

Math Problem of the Month - February 2019

Sequences occur in all branches of Mathematics. You may know some famous recursive sequences like the Fibonacci numbers: $F_0 = 0, F_1 = 1$ and for $n \geq 0$, $F_{n+2} = F_{n+1} + F_n$. This is an example of a **linear** recursive sequence. Less common examples are **nonlinear**.

We'll define a nonlinear recursive sequence.

First, let $W_0 = 0, W_1 = 1, W_2 = 1, W_3 = -3, W_4 = 1$.

Now, define the following recursion for all $m > n$,

$$W_{m+n}W_{m-n} = W_{m+1}W_{m-1}W_n^2 - W_{n+1}W_{n-1}W_m^2.$$

This turns out to produce a well-defined sequence (there's no need to prove this).

Now **prove** that for all $n \in \mathbb{N}$, $W_n \in \mathbb{Z}$. To start, you may wish to gain intuition by calculating W_5 .

Put your solution into the dropbox for the Math Problem of the Month!