Stephanie Erickson, 2009/2010, “An Applet to Illustrate Curved Geometry”

One of the fundamental components of Einstein's theory of general relativity is the concept of curved space-time; while classical theories treat gravity as a force, general relativity proposes that objects follow geodesics, or straight lines, through a curved space-time. In addition to being one of the most fundamental aspects of general relativity, curved space-time is also one of the hardest to understand. Many students attempt to understand curved space-time, and the simpler concept, curved space, through embedding diagrams, which represent non-Euclidean geometries by showing them as curved surfaces in higher-dimensional Euclidean spaces. Although embedding diagrams are extremely useful, because they allow students to use their physical intuition to understand the functioning of a certain geometry, they can be misleading; embedding diagrams can encourage students to believe that curved space must exist in some higher-dimensional Euclidean space. To combat this misconception, we have created an applet to simulate two-dimensional space of constant positive curvature, which is mathematically equivalent to the surface of a sphere. The applet includes features that illustrate the curvature of the space: the viewer can draw shapes and then move around within the space to observe how those shapes appear to distort. The applet is available on the internet and is accompanied by several tutorials that guide the user through activities that show how curved space is different from flat space.

Amanda (Stevie) Bergman, 2008/2009, “Phase Portraits of Inflationary Dynamics”

Inflation is the theory that governs the earliest moments of the universe, the first fraction of a second after the big bang. Inflation purports that, just after the big bang there was a scalar field (dubbed the inflation field) that caused the universe to undergo exponential expansion. Since its conception, the inflationary theory has been very widely studied, however not a lot has been done with non-minimally coupled inflation. Inflation is non-minimally coupled (rather than minimally coupled, which is what is generally studied) when the inflation field is coupled to gravity, meaning that the motion of the scalar field and the local curvature of space-time are dependent on each other. In this work I use phase portraits, and a specific type of phase portrait called a Poincare map, to study the
behavior of a non-minimally coupled inflation. In essence, this thesis works to determine whether or not certain models of inflation that lead to inflation with a minimally coupled scalar field, would produce inflation when that field is non-minimally coupled to gravity, and vice versa. We determine this using several different plotting techniques in Mathematica that clarify if and where the universe instantiations (trajectories) on the phase portraits and Poincare maps indicate inflation, if at all. These techniques prove especially useful in the non-minimally coupled case where the equations of motion are complicated. In this study, we focus on two models, the widely studied m^2 \phi^2 model and a warped brane inflation model that was shown to lead to inflation when the inflation is non-minimally coupled.

We found that for the m^2\phi^2 model there is inflation in the minimally coupled case, but not when the inflation is non-minimally coupled to gravity. With the string theory potential, I found that there is inflation in both the minimally and non-minimally coupled cases.

![Phase portrait of a non-minimally coupled brane inflation model](image-url)