

The Symmetry Energy at Sub- and Supersaturation Densities in Heavy Ion Collisions

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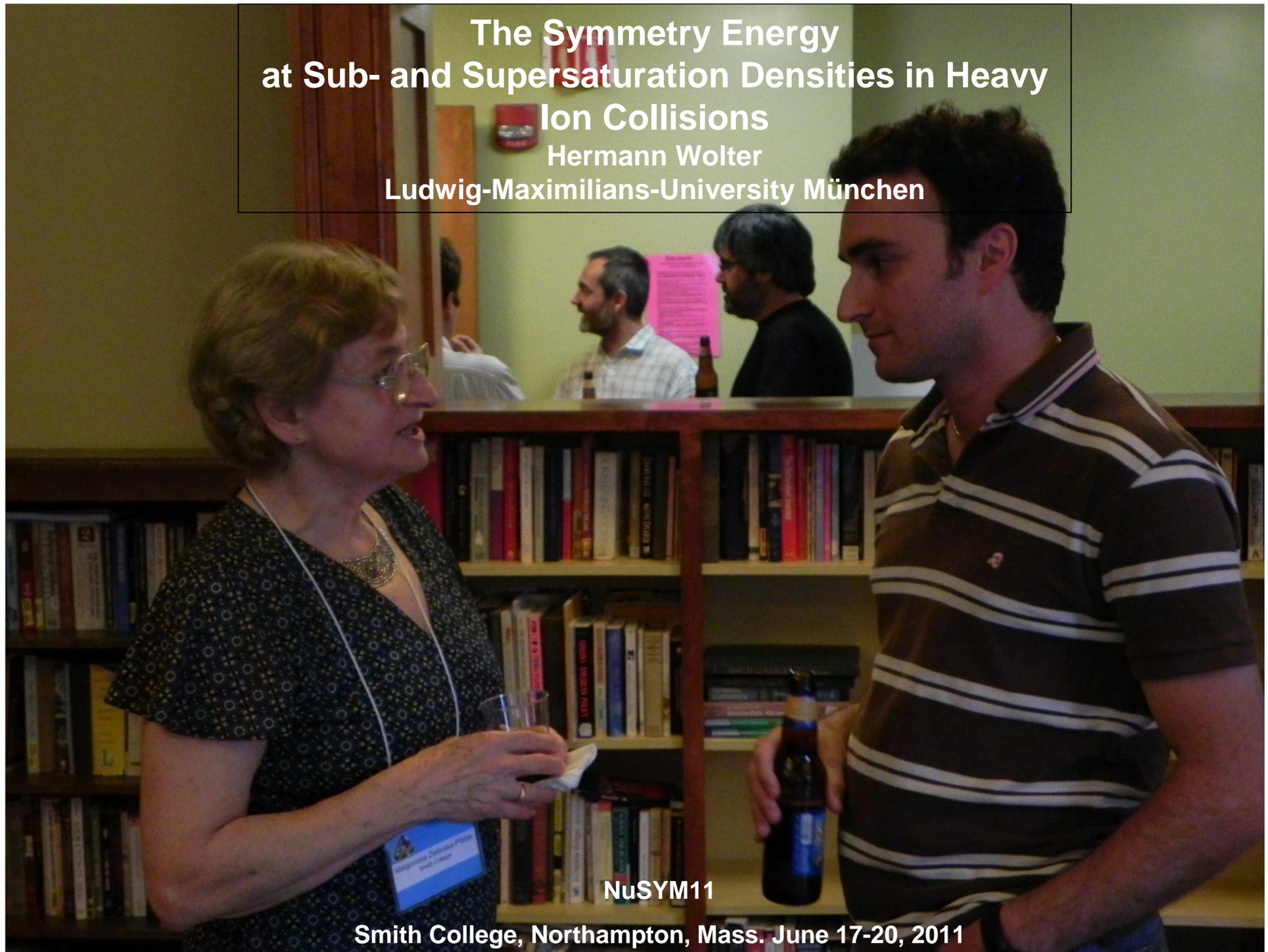


NuSYM11

Smith College, Northampton, Mass. June 17-20, 2011



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Points to discuss

- explore the symmetry over a large range of densities with many probes
- a satisfactory consistent picture (between different observables and between different groups) has not yet been achieved: it is better at subsaturation density but still lacking at suprasaturation)
- the momentum dependence of the symmetry energy (difference in neutron and proton effective masses) has to be considered for a reliable interpretation (and is still controversial)

I report on work of the Catania reaction theory group and associated members:

Massimo Di Toro, Maria Colonna, Vincenzo Greco, Joseph Rizzo, Carmelo Rizzo, Valentina Giordano, Lab. Naz. del Sud, Catania

Virgil Baran, University of Bucharest, Romania

Malgorzata Zielinska-Pfabe, Smith Coll., USA

Theodoros Gaitanos, Univ. of Giessen

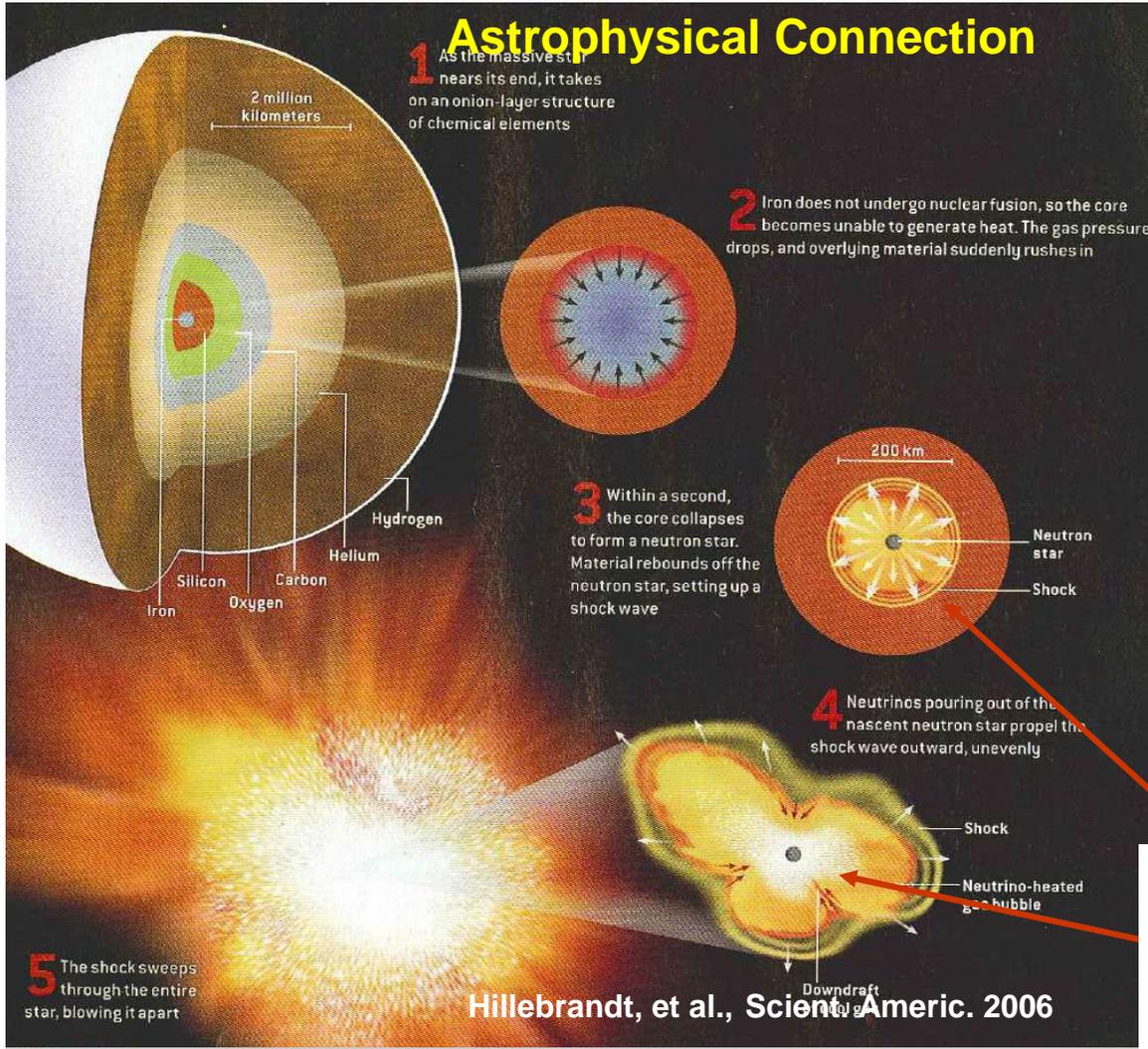
Vaia Prassa, Georgios Lalazissis, Aristotle Univ. Thessaloniki, Greece

Tatiana Mikhailova, Dubna, Russia

Plan:

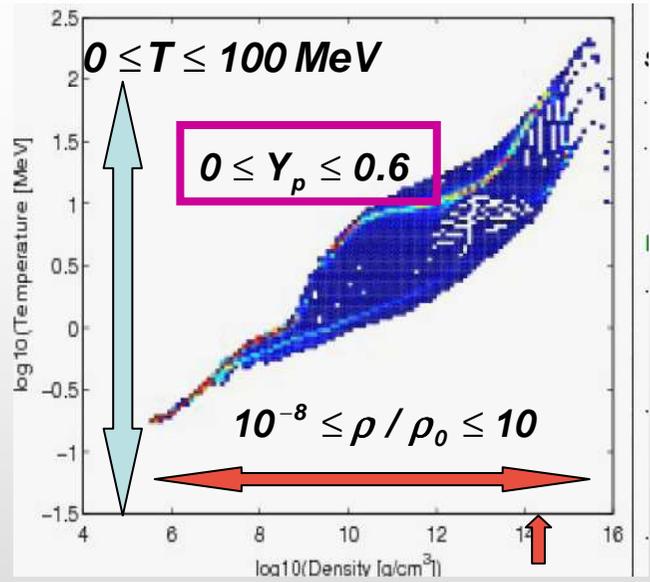
- Relevance of symmetry energy to astrophysics
- Overview over different probes at different energies
- Analysis of important and illustrative cases

Astrophysical Connection



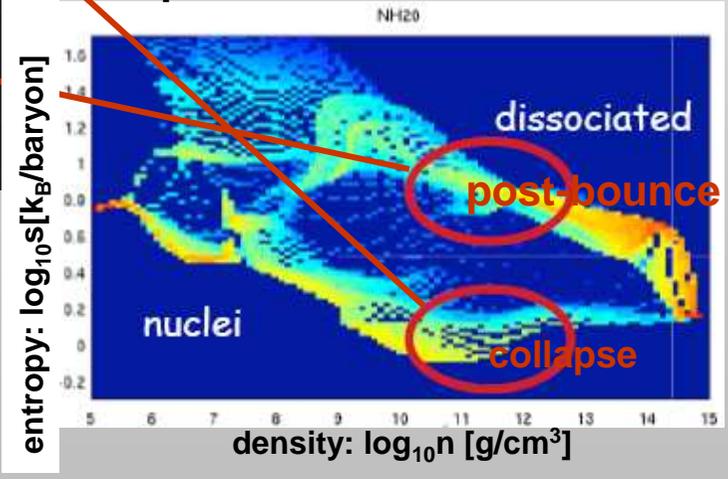
Hillebrandt, et al., *Scientific American*, 2006

Ranges of density, temperature, asymmetry in SN event



Liebrandt, et al., NIX XI, 2010

Composition of the matter



Symmetry energy plays an essential role in astrophysics

not uniform, but light and heavy clusters (nuclei embedded in the medium)

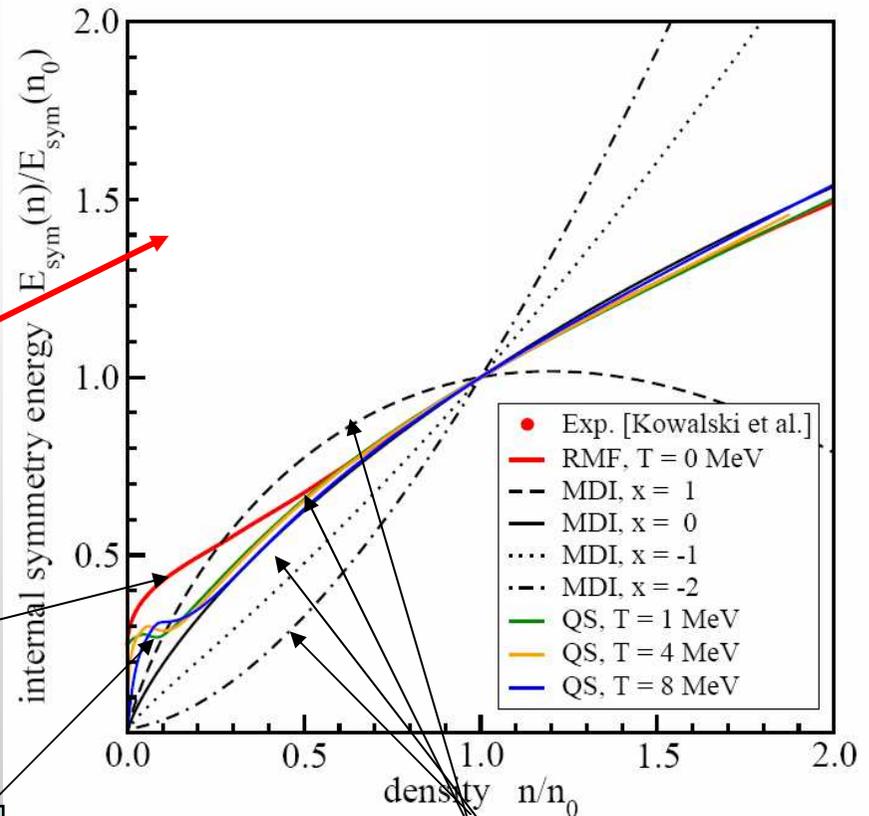
EOS for use in Supernovae (and neutron stars):
Cf. talk of Maria Voskresenskaya (+ S. Typel, G. Röpke, et al.)

Thermal Green Fct
(quantumstatistical) model for
medium dependence of clusters
+ gen RMF model (with explicit
cluster degrees of freedom)
+ heavy nucleus in the medium (in
Wigner-Seitz cell approx)

Comparison to
low low
density SE
determined in
heavy ion
collisions
(Natowitz et al.,
PRL 104 (2010))

Single nucleus
approx. (Wigner-
Seitz), RMF

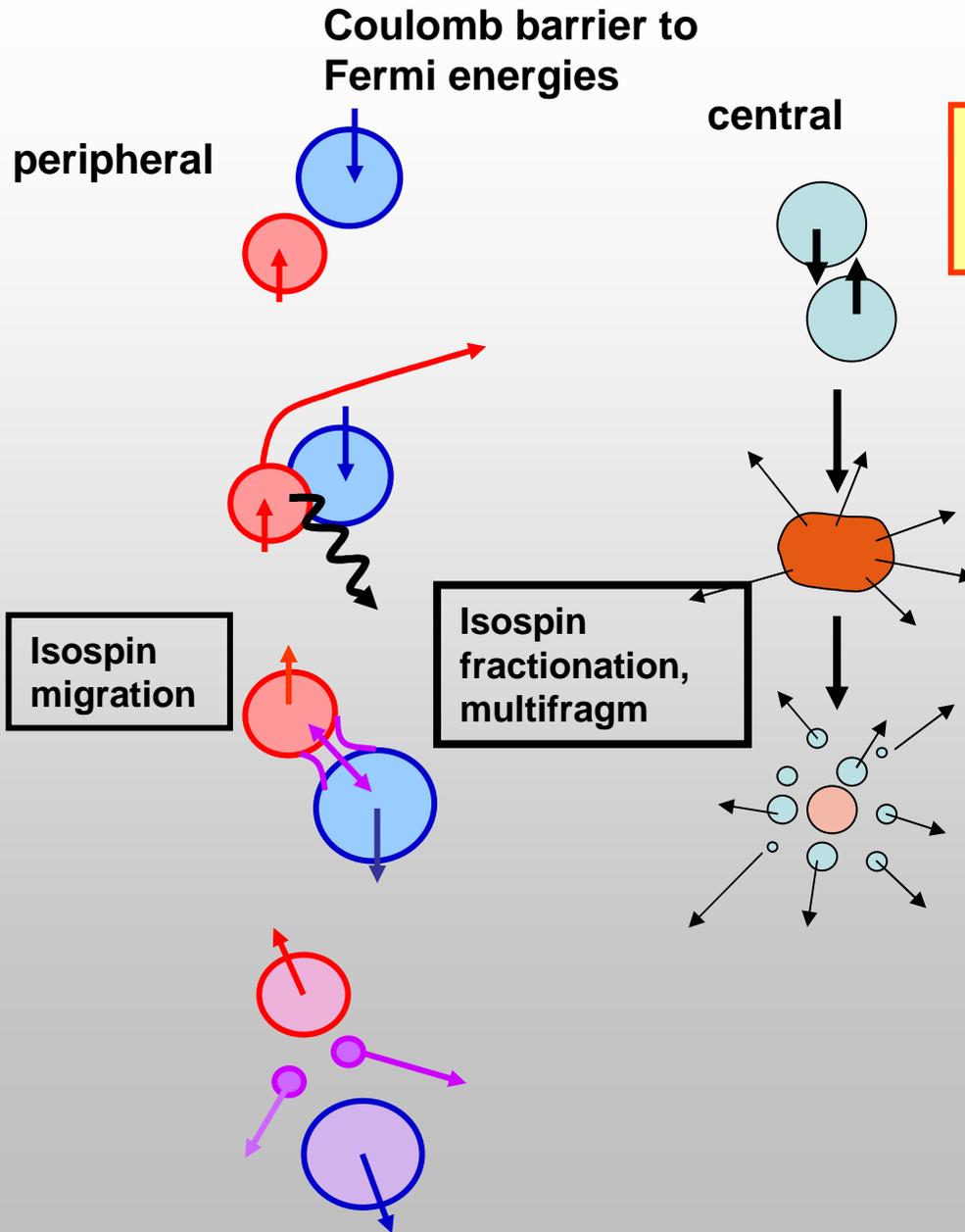
Quantum
Statistical model ,
T=1,4,8 MeV)



Parametrization of nuclear
symmetry energy of different
stiffness (momentum dependent
Skyrme-type) (B.A. Li)

The symmetry energy is finite at low density and temperatures!

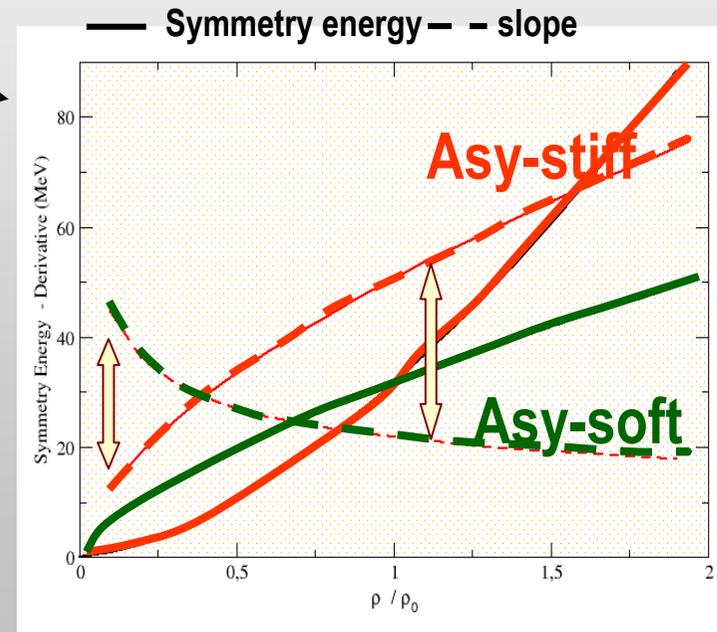
Reaction Mechanisms in Heavy Ion Collisions



Proton and neutron currents

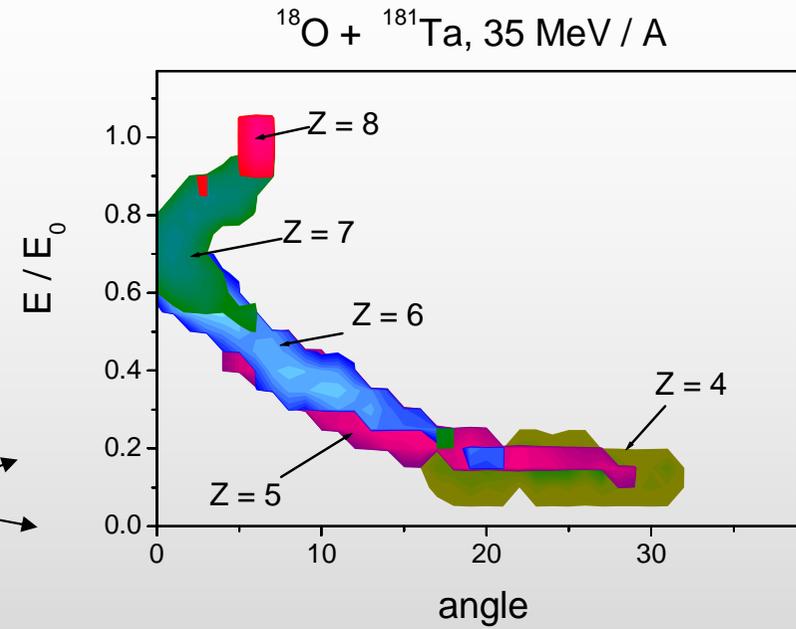
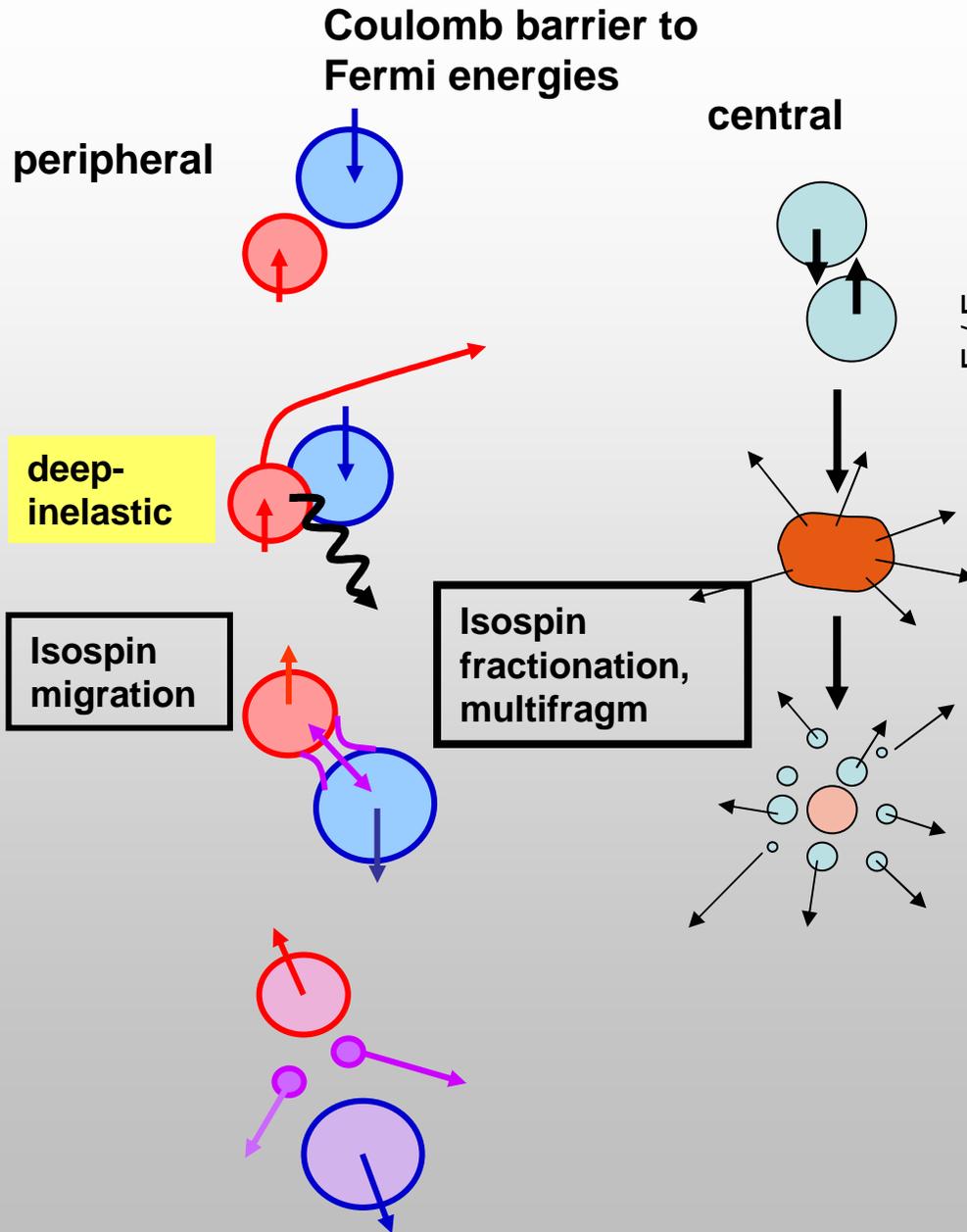
$$j_n - j_p \propto E_{sym}(\rho) \nabla I + \frac{\partial E_{sym}(\rho)}{\partial \rho} I \nabla \rho$$

↑ "Diffusion" ↑ "Drift"

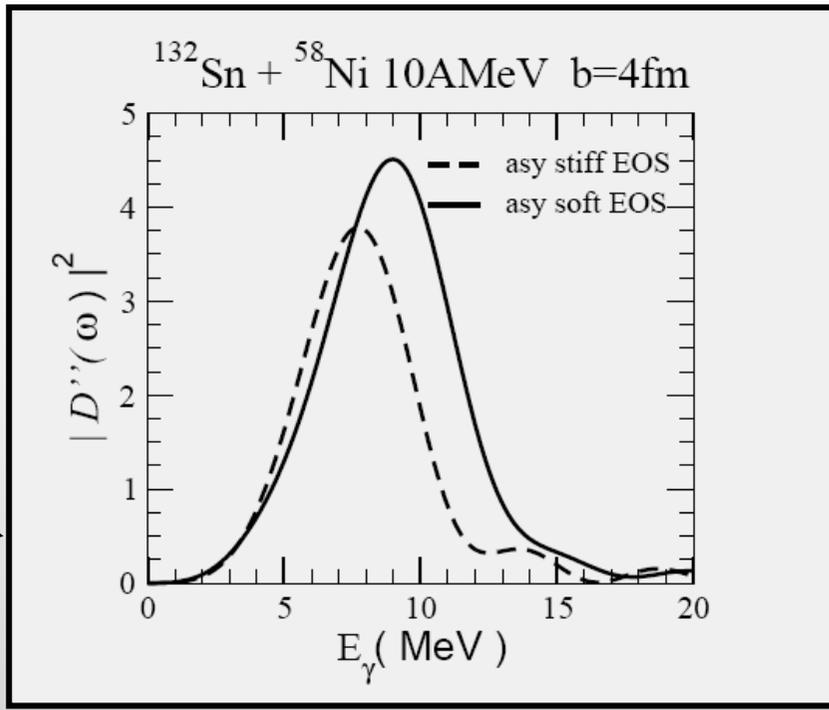
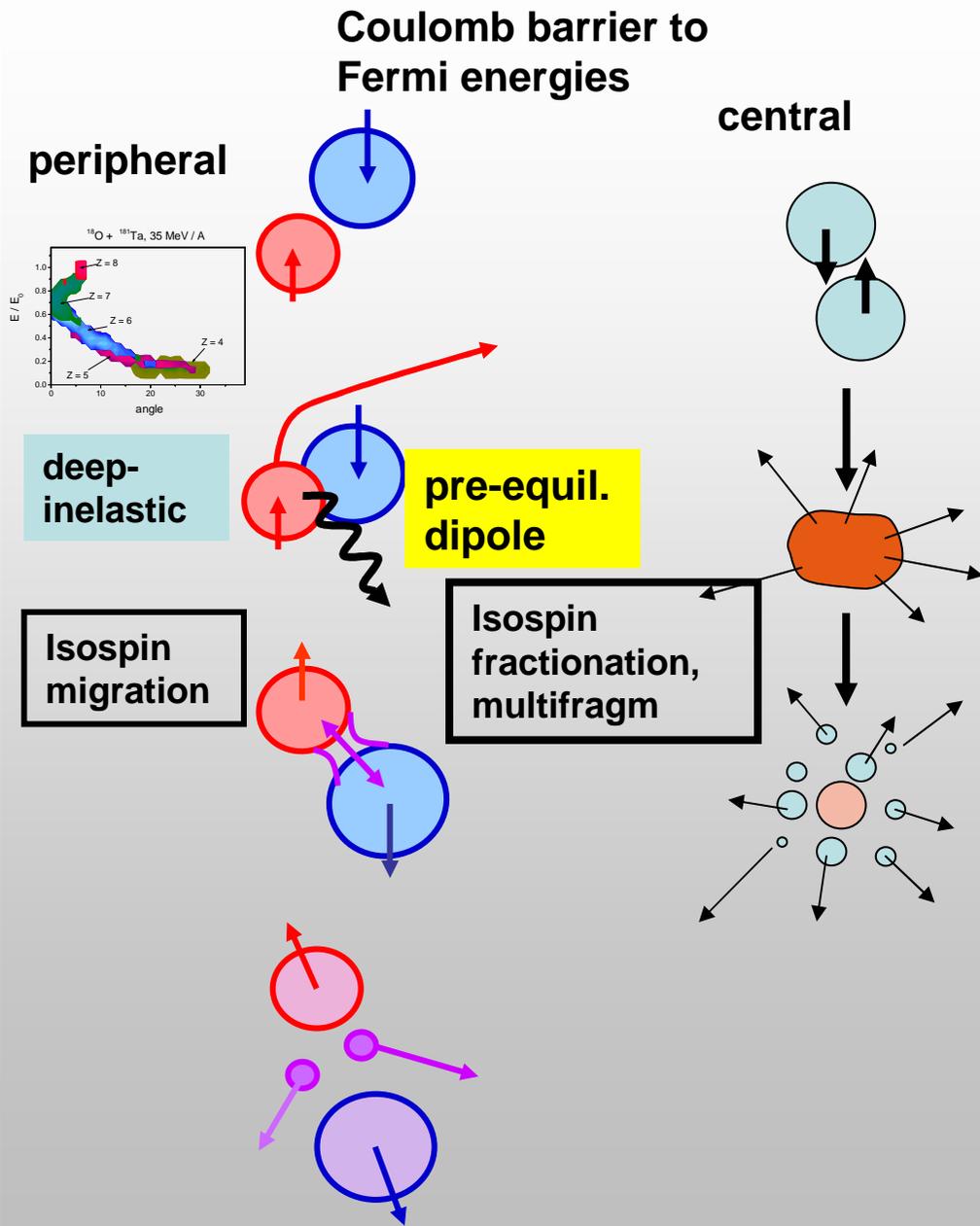


Sensitive to Sym. Energy and slope depending on observable

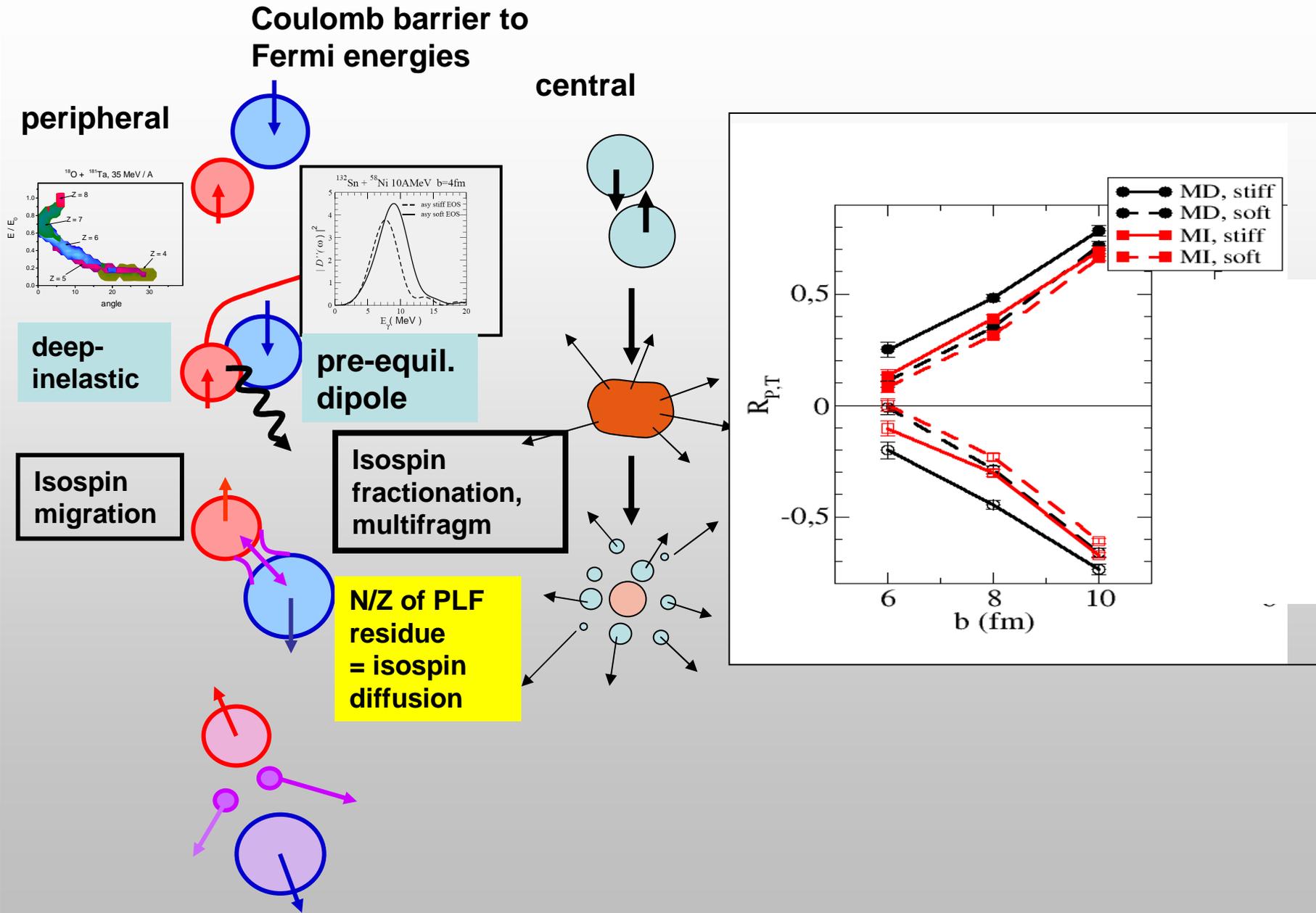
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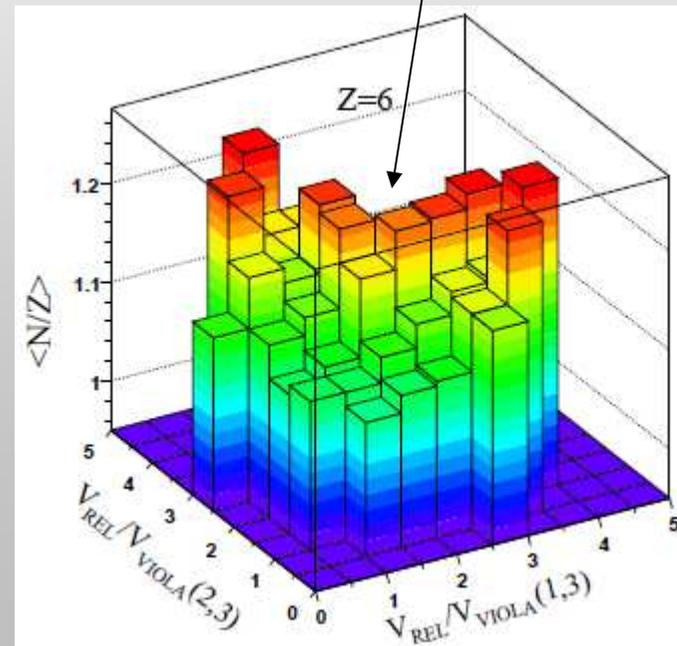
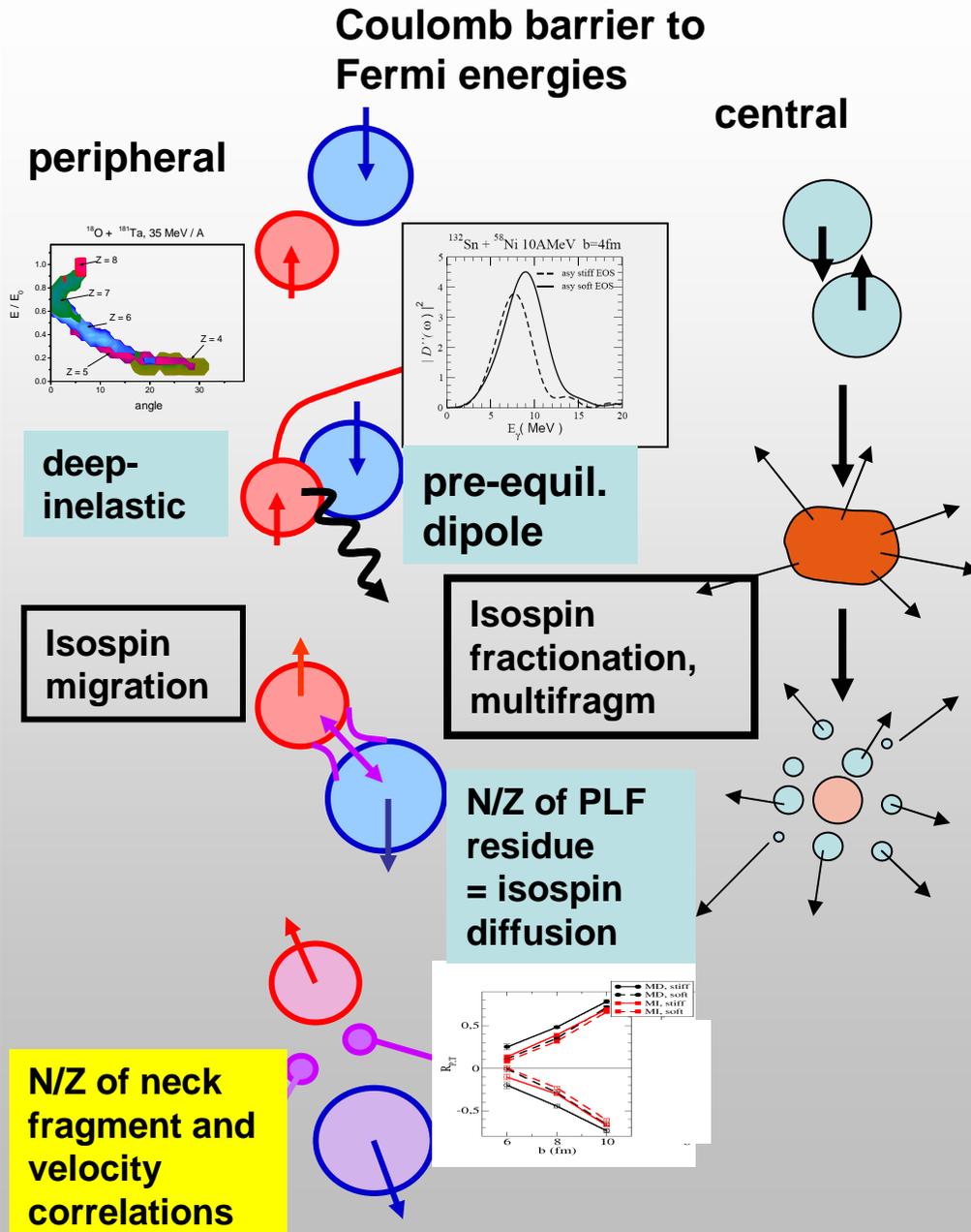
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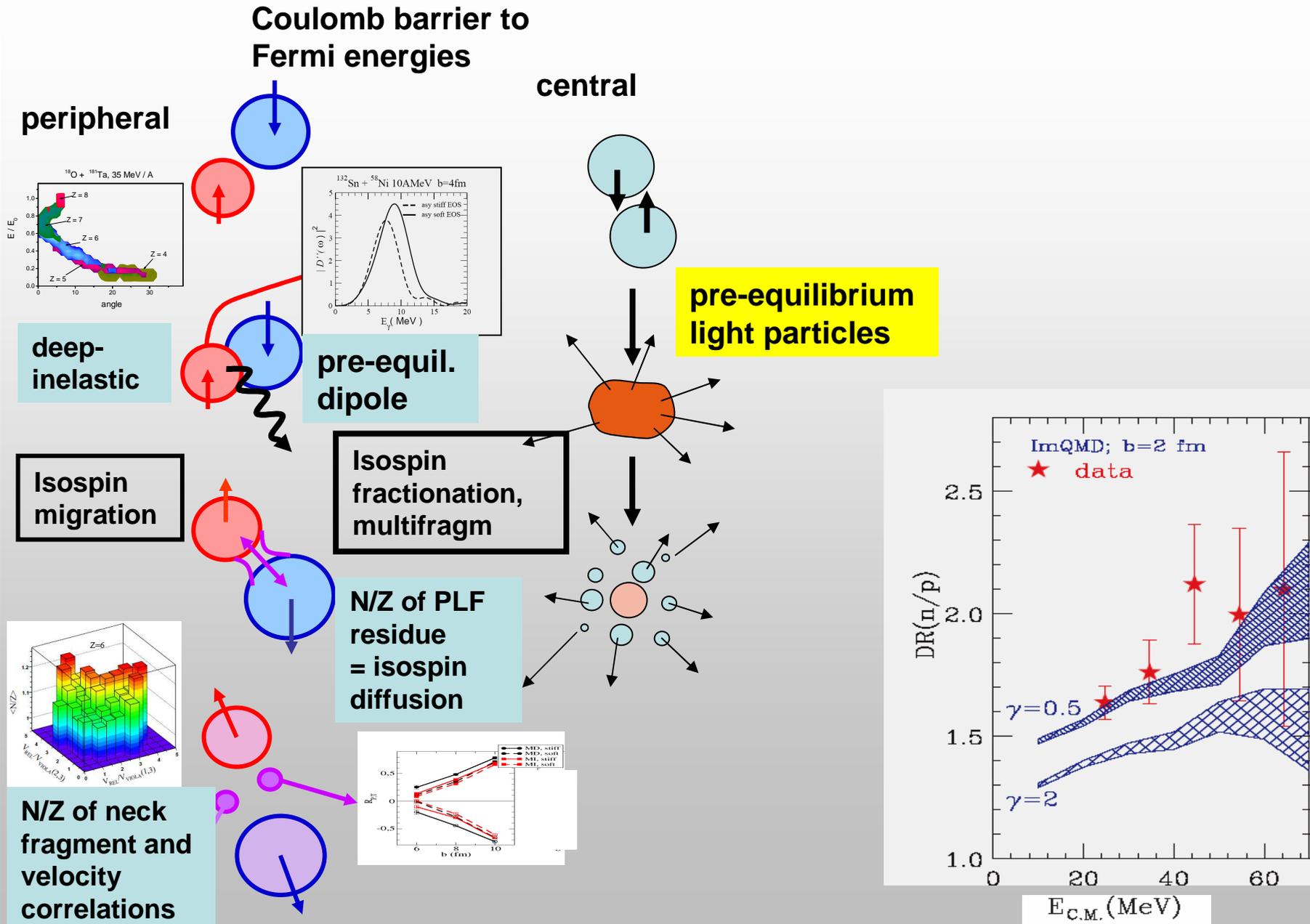
Reaction Mechanisms in Heavy Ion Collisions



Reaction Mechanisms in Heavy Ion Collisions



Reaction Mechanisms in Heavy Ion Collisions

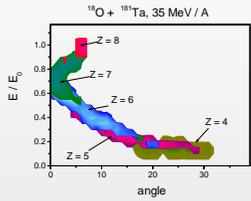


Reaction Mechanisms in Heavy Ion Collisions

Coulomb barrier to Fermi energies

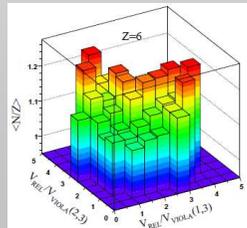
Relativistic energies

peripheral

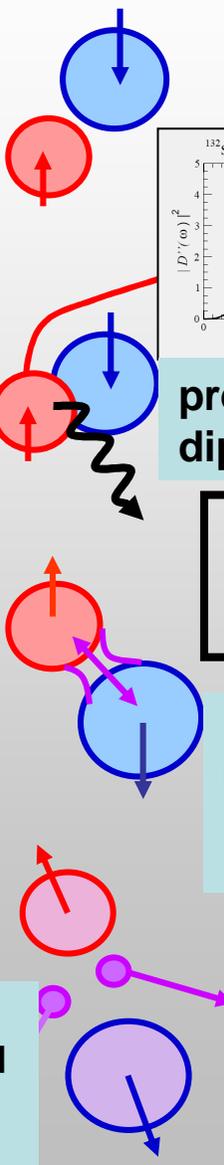


deep-inelastic

Isospin migration



N/Z of neck fragment and velocity correlations

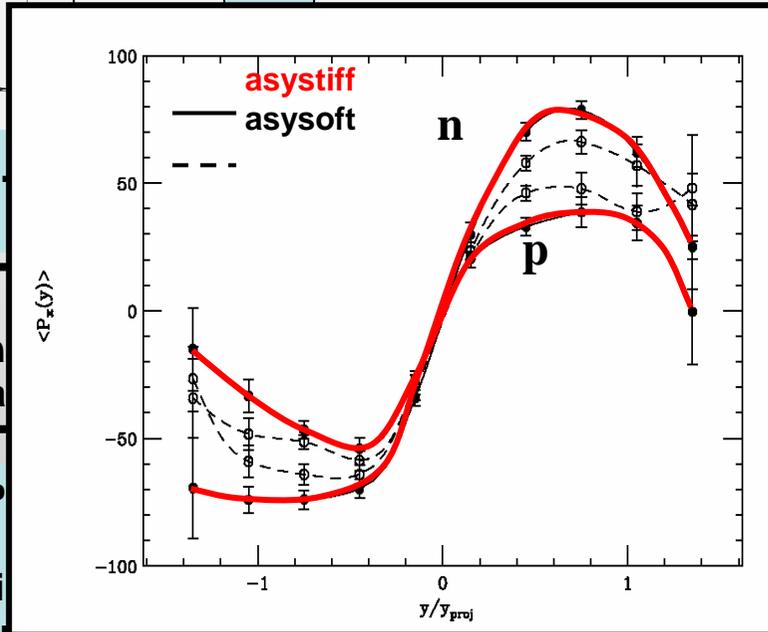
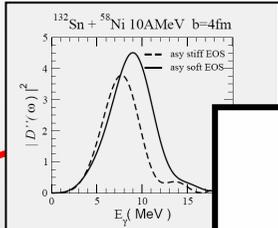
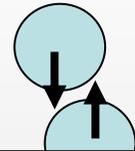


pre-equil. dipole

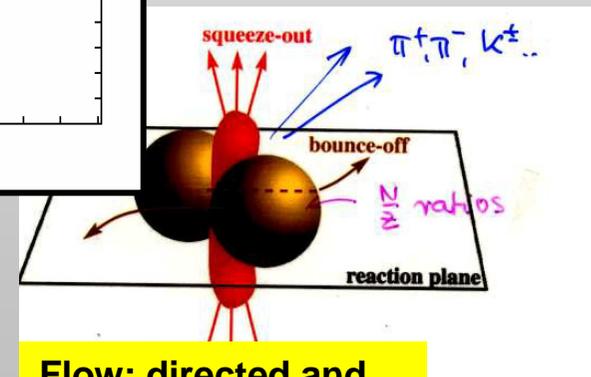
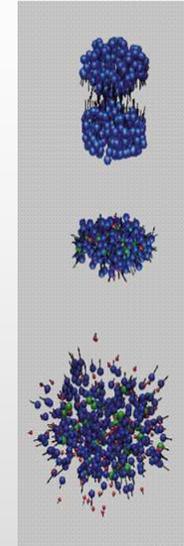
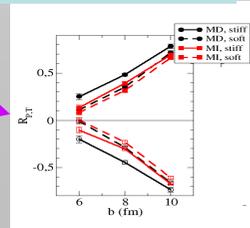
Isospin fraction multifra

N/Z of P residue = isospin diffusion

central



N/Z ratio of IMF's



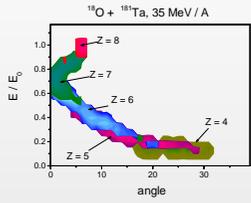
Flow: directed and elliptic

Reaction Mechanisms in Heavy Ion Collisions

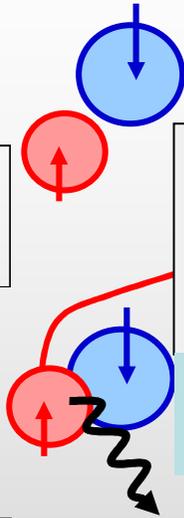
Coulomb barrier to Fermi energies

Relativistic energies

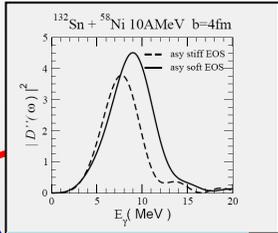
peripheral



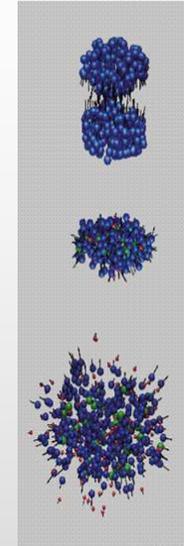
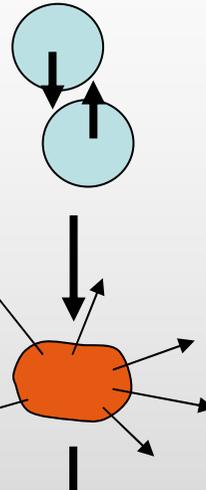
deep-inelastic



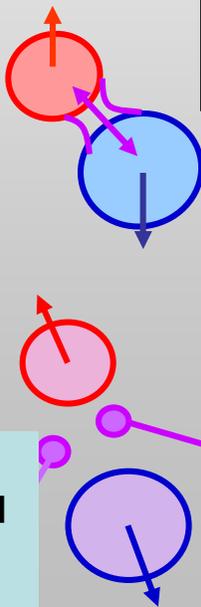
pre-equil. dipole



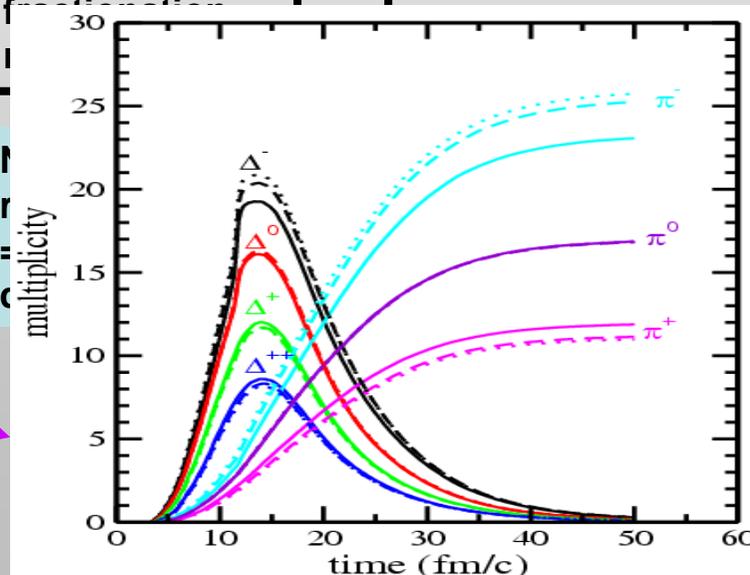
central



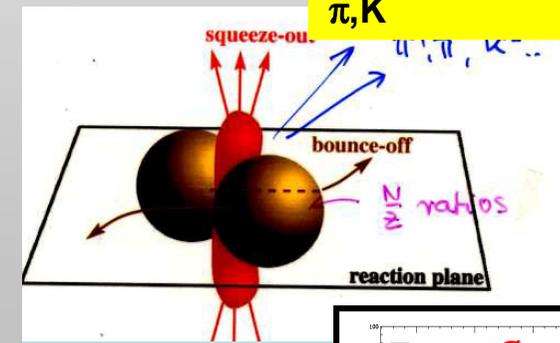
Isospin migration



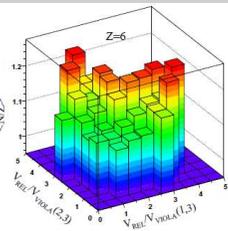
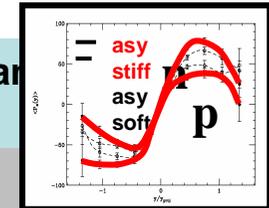
Isospin



Production of particles, π, K



Flow: directed and elliptic



N/Z of neck fragment and velocity correlations

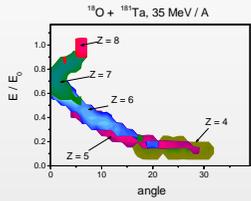
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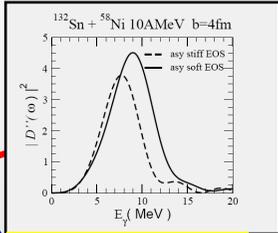
Relativistic energies

peripheral

central



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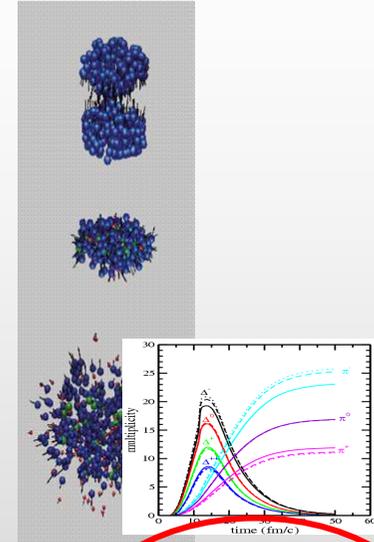


pre-equil. dipole

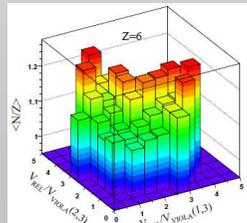
Isospin migration

Isospin fractionation, multifragm

pre-equilibrium light particles

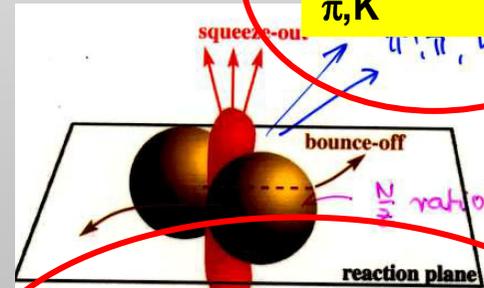


Production of particles, π, K

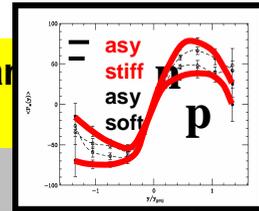


N/Z of PLF residue = isospin diffusion

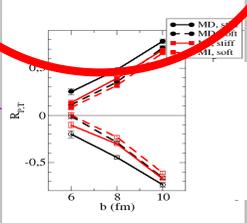
2. N/Z ratio of IMF's



Flow: directed and elliptic



5. N/Z of neck fragment and velocity correlations



Transport Description of Heavy Ion Collisions

one-body phase space density: $f(\mathbf{r}, \mathbf{p}; t)$

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \nabla f - \nabla U \nabla_p f = I_{coll} [f, \sigma] + \delta I_{fluc}$$

Vlasov eq.; mean field

2-body hard collisions

$$= \frac{1}{(2\pi)^{3/2}} \iiint dp_2 dp_3 dp_4 v_{12} \frac{d\sigma}{d\Omega_{12 \rightarrow 34}} \delta(p_1 + p_2 - p_3 - p_4) [(1-f)(1-f_2)f_3f_4 - f f_2(1-f_3)(1-f_4)]$$

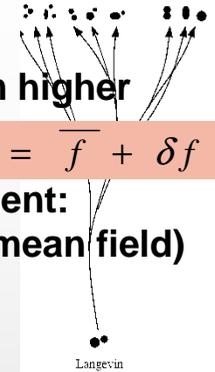
loss term

gain term

Fluctuations from higher order corr.;

$$f = \bar{f} + \delta f$$

stochastic treatment: SMF (stochastic mean field)



Relativistic BUU eq.

$$\left[\mathbf{p}^{*\mu} \partial_{\mu}^{(r)} + (\mathbf{p}^*_{\nu} \mathbf{F}^{\mu\nu} + m^* \partial_{(r)}^{\mu} m^*) \partial_{\mu}^{(p^*)} \right] f(\mathbf{r}, \mathbf{p}^*) = I_{coll}$$

effective mass $m^* = m - \Sigma_s$

Kinetic momentum $\mathbf{p}^*_{\mu} = \mathbf{p}_{\mu} - \Sigma_{\mu}$

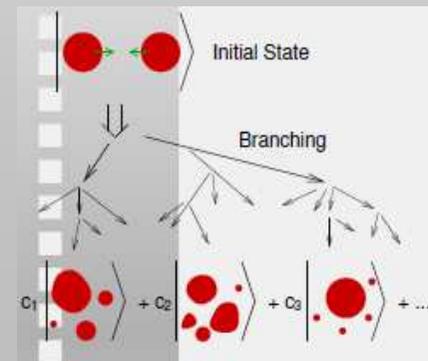
Field tensor $\mathbf{F}^{\mu\nu} = \partial^{\mu} \Sigma^{\nu} - \partial^{\nu} \Sigma^{\mu}$

Quantum molecular dynamics QMD

$$|\Phi(\mathbf{Z})\rangle = \det_{ij} \left[\exp \left\{ -v \left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{v}} \right)^2 \right\} \chi_{\alpha_i}(j) \right]$$

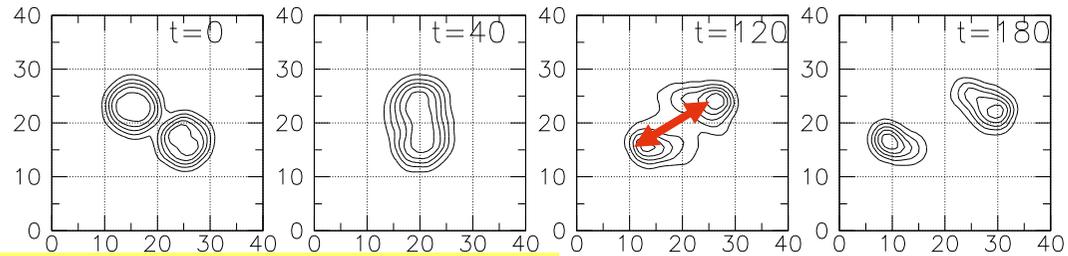
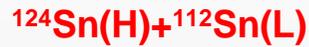
Stochastic equation of motion for the wave packet centroids \mathbf{Z} :
(stochastic NN collisions)

$$\frac{d}{dt} \mathbf{Z}_i = \{ \mathbf{Z}_i, \mathcal{H} \}_{PB} + \Delta \mathbf{Z}_i(t) + (\text{NN collisions})$$



Isospin diffusion

isospin transport through „neck“ in peripheral collisions



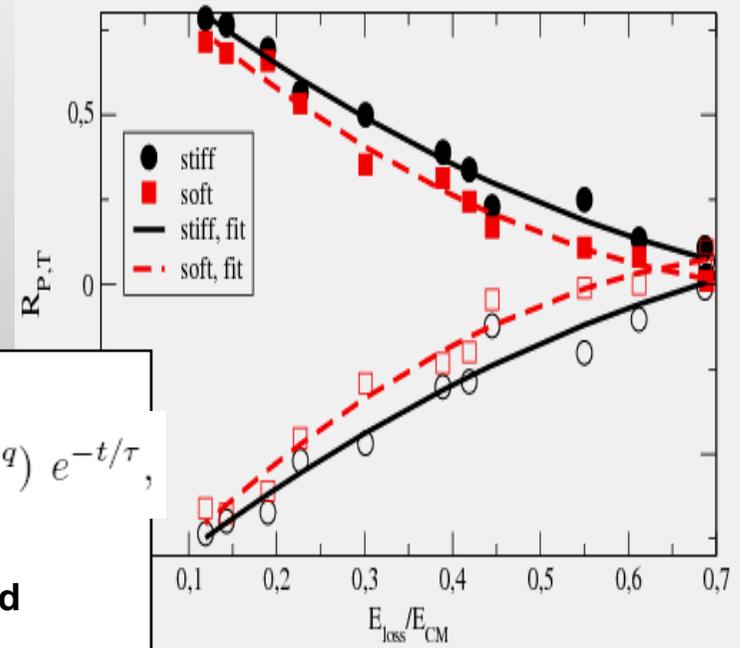
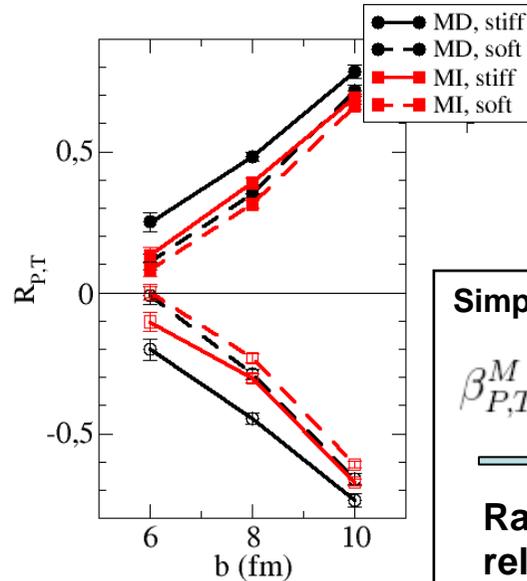
Imbalance (or Rami, transport) ratio:

β asymmetry of residue (i=PLF,TLF)
(also for other isospin sens.quantities)

$$R_i = \frac{\beta_i^{mix} - \frac{1}{2}(\beta_i^{HH} + \beta_i^{LL})}{\frac{1}{2}(\beta_i^{HH} - \beta_i^{LL})}$$

Limiting values:

$R=0$ complete equilibration
 $R=+1$, complete transparency



Simple equil. model

$$\beta_{P,T}^M = \beta^{eq} + (\beta^{H,L} - \beta^{eq}) e^{-t/\tau}$$

$$\longrightarrow R_{P,T} = \pm e^{-t/\tau}$$

Ratio det. by interact. and relax. times

more equilibration (lower R) for longer interaction time ~ correlation with total energy loss

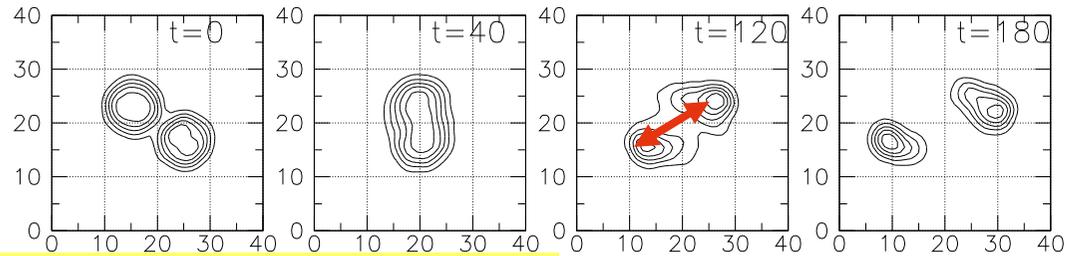
Isospin transport

isospin transport through „neck“ in peripheral collisions

$^{124}\text{Sn}(H)+^{112}\text{Sn}(L)$

Imbalance (or Rami, transport) ratio:

β asymmetry of residue (i=PLF,TLF)
(also for other isospin sens.quantities)



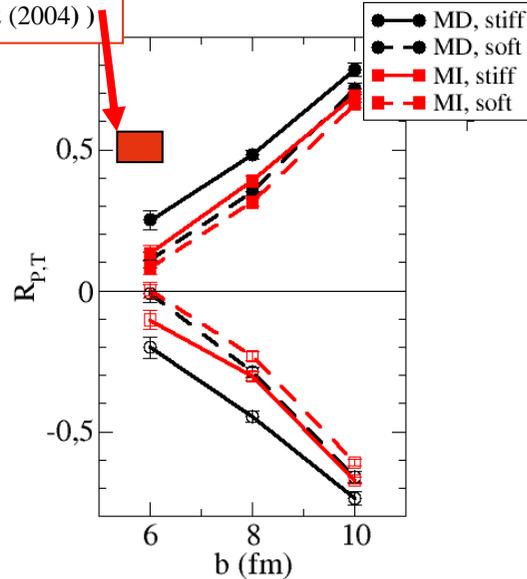
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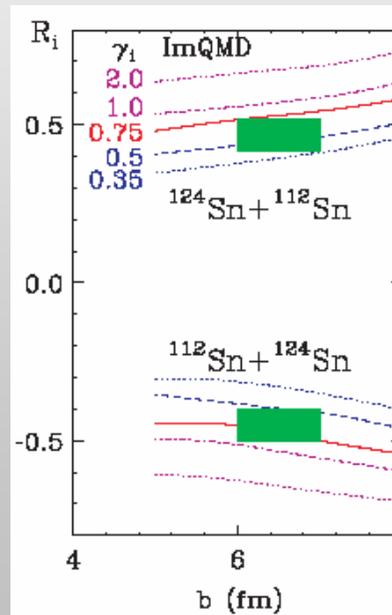
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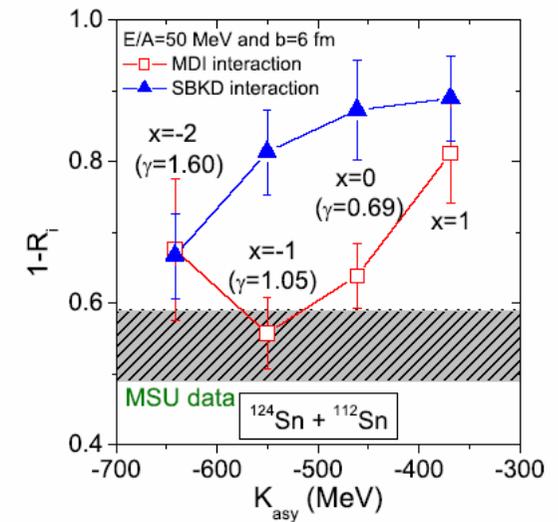
experimental data
(B. Tsang et al.
PRL 92 (2004))



J.Rizzo, et al., Nucl. Phys. A806 (2008) 79



M.B. Tsang, et al., PRL 102 (2008)



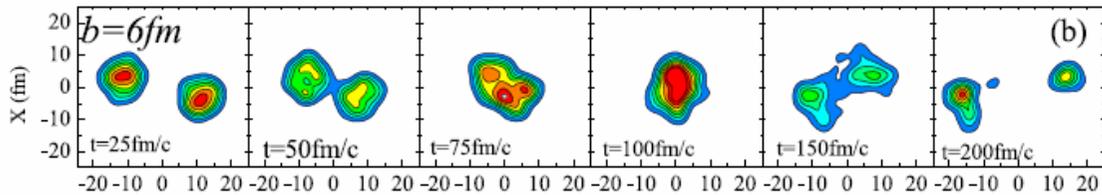
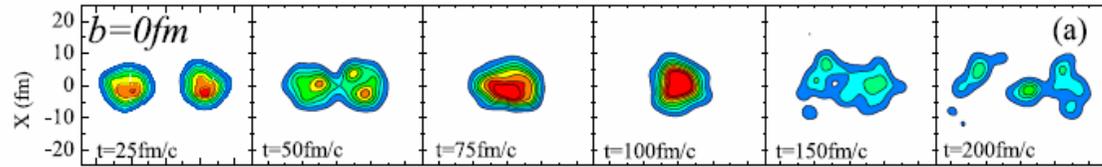
L.W.Chen, C.M.Ko, B.A.Li, PRL 94, 032701 (2005)

points toward a moderately stiff ($\gamma \sim 1$) SE, but disagreement in detail

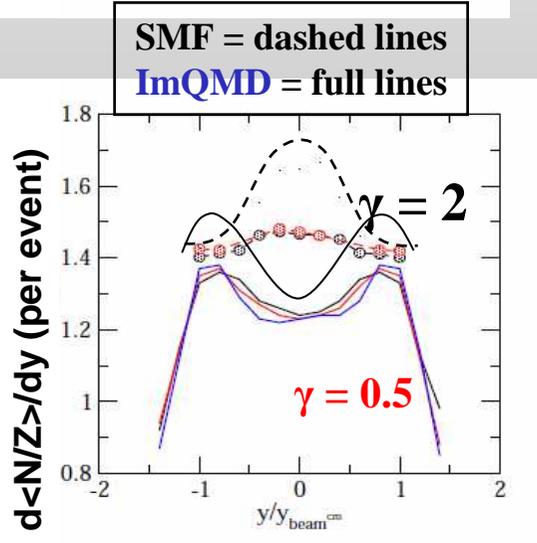
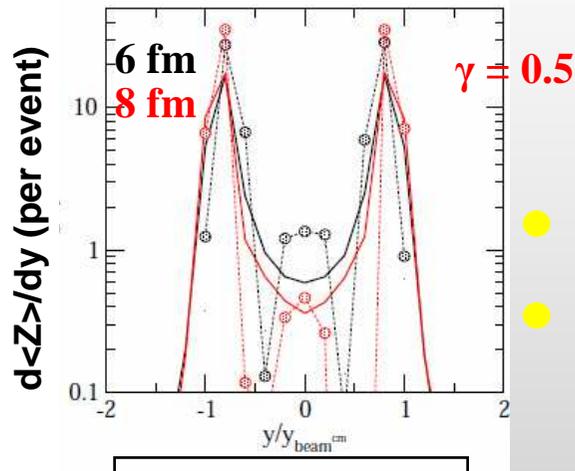
Analysis of differences BNV – QMD

→ more 'explosive' dynamics:
more 'transparency'

ImQMD calculations, $^{112}\text{Sn} + ^{112}\text{Sn}$, 50 A MeV



Y.Zhang et al., arXiv:1009.1928

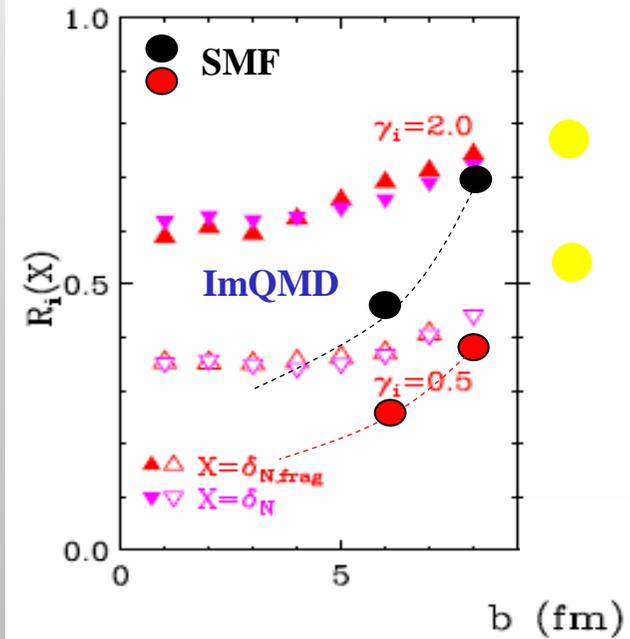


SMF = dashed lines
ImQMD = full lines

Much less isospin migration in ImQMD,
Other sources of dissipation: Fragment emission, fluctuation?

→ Less dependence on impact parameter.

Similar conclusions in comparison with Antisymmetrized Mol. Dynamics (AMD): Colonna, Ono, Rizzo, PRC (2011)



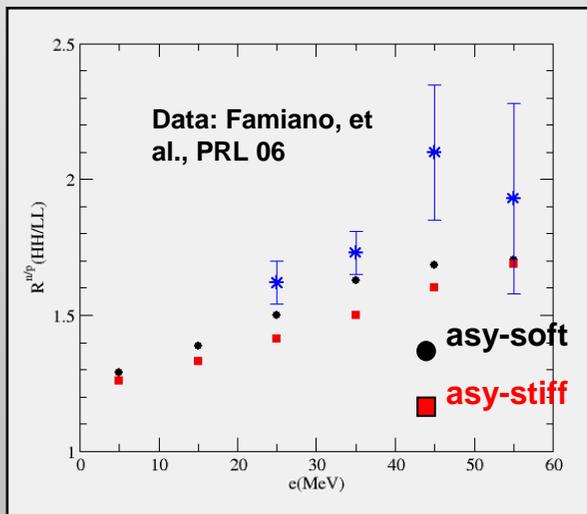
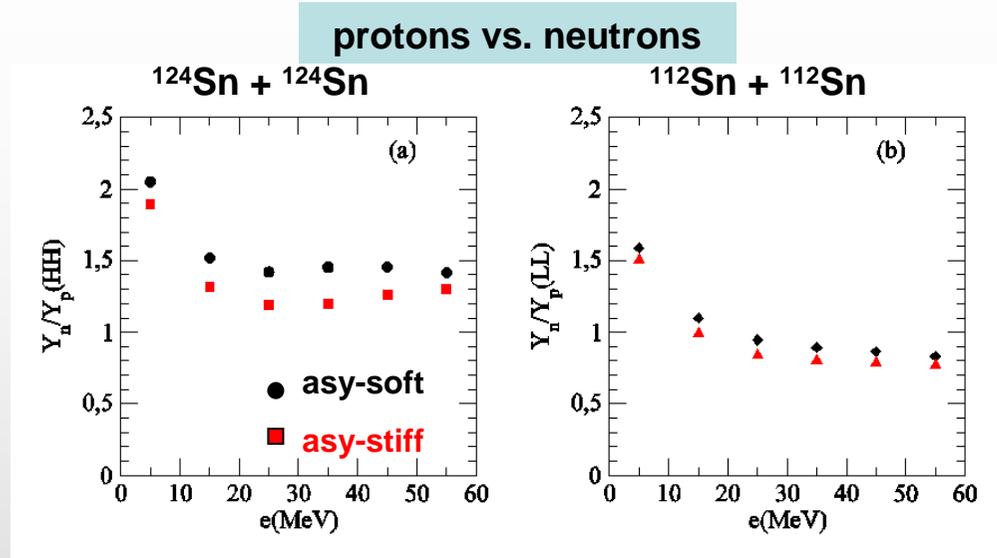
Model dependence of imbalance ratio!

Central collisions at Fermi energies: Ratios of emitted pre-equilibrium particles

Early emitted neutrons and protons reflect difference in potentials in expanded source, esp. ratio $Y(n)/Y(p)$.

more emission for asy-soft, since symm potential higher

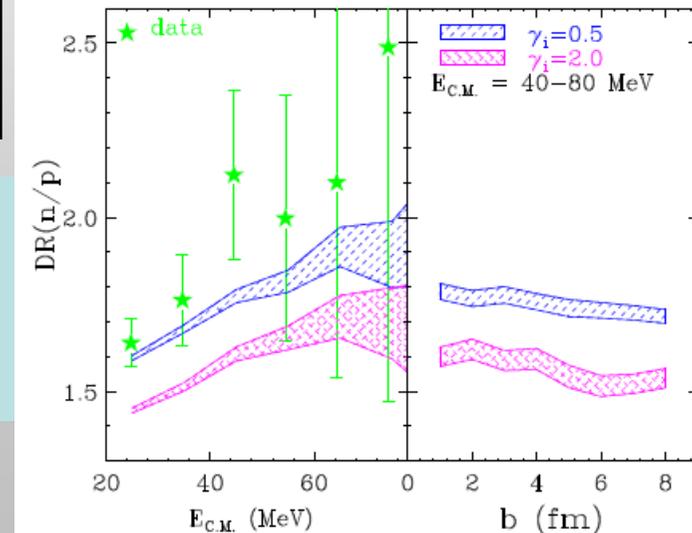
„Double Ratios“ $\frac{^{124}\text{Sn} + ^{124}\text{Sn}}{^{112}\text{Sn} + ^{112}\text{Sn}}$



SMF simulations, V.Baran 07

softer symmetry energy closer to data

qualitatively as seen in ImQMD, but quantitatively weaker dependence on SymEn

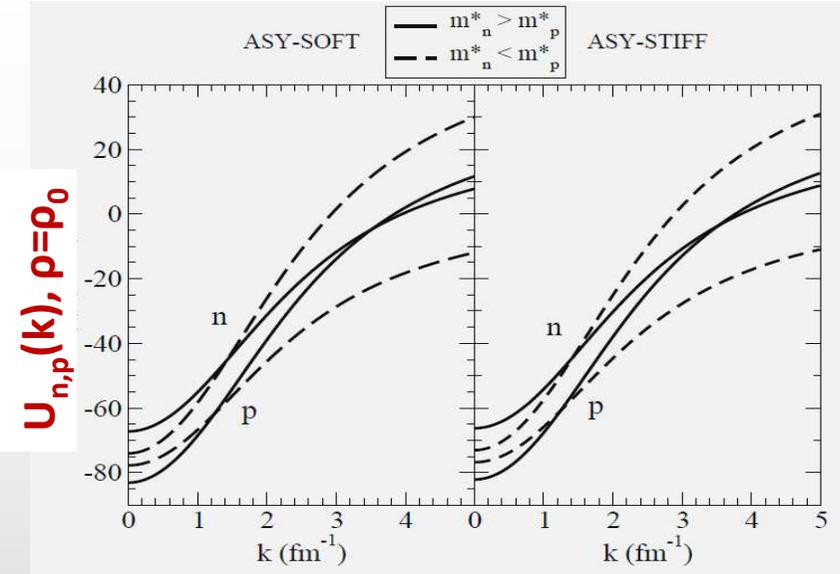


Y. Zhang, et al., arXiv (2011)

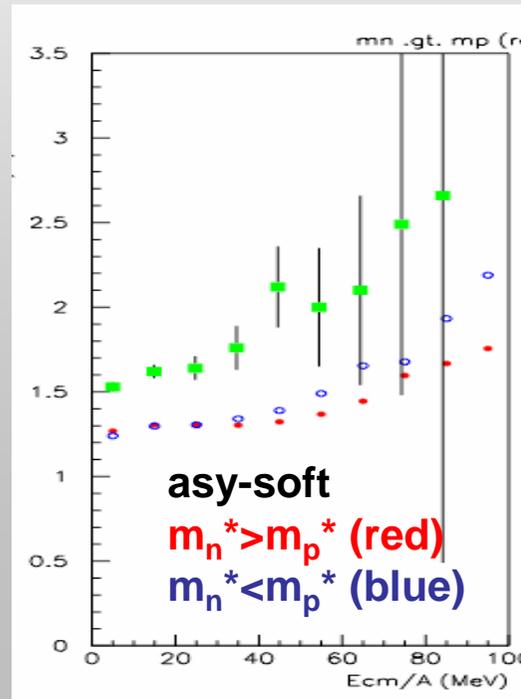
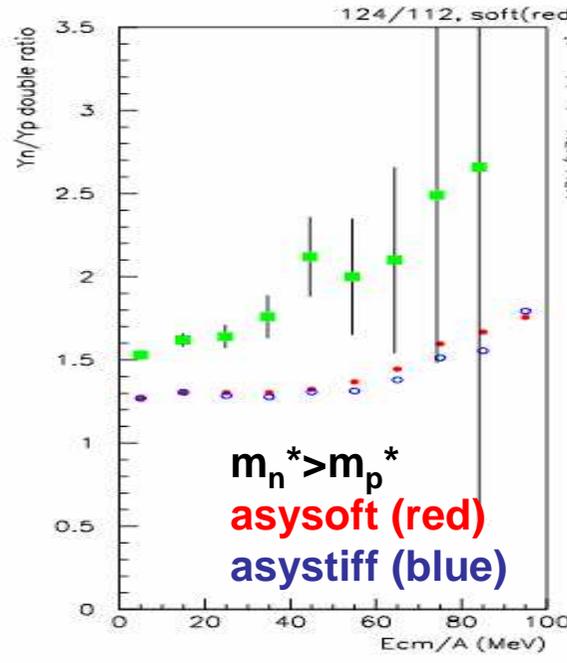
Momentum Dependence and Effective Mass Splitting

$$\frac{m_q^*}{m} = \left[1 + \frac{m}{\hbar^2 k} \frac{\partial U_q}{\partial k} \right]^{-1}$$

At high momentum m^* splitting effect larger than asy-stiffness



Sn+Sn, 50 A MeV

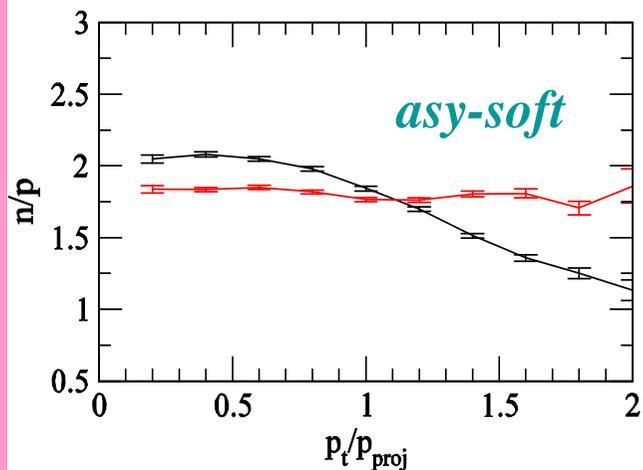
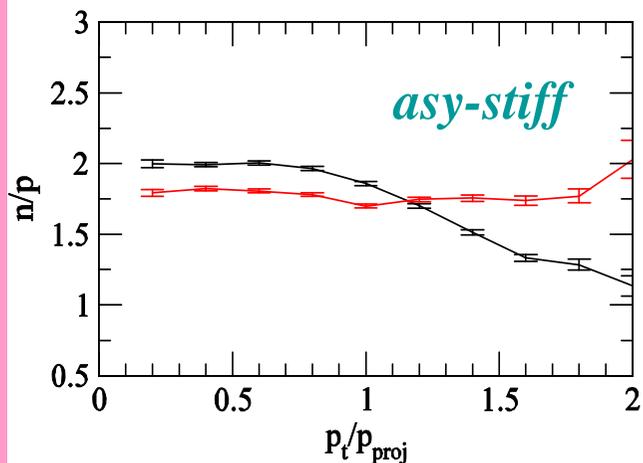


Effect of momentum dependence of symmetryenergy as large as effect of asy-EOS itself.

Momentum dependence here rather strong

Pre-equilibrium nucleon and light cluster emission at higher energy

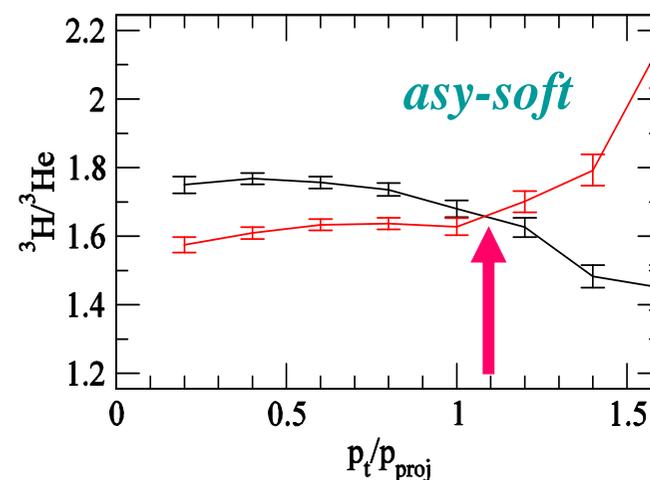
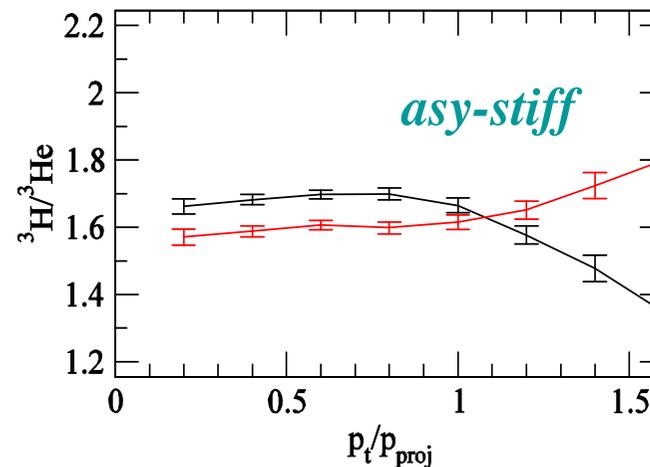
n/p ratio yields



$^{197}\text{Au}+^{197}\text{Au}$
600 A MeV
 $b=5$ fm,
 $y(0)\leq 0.3$

• $m_n^* > m_p^*$
• $m_n^* < m_p^*$

Light isobar $^3\text{H}/^3\text{He}$ yields



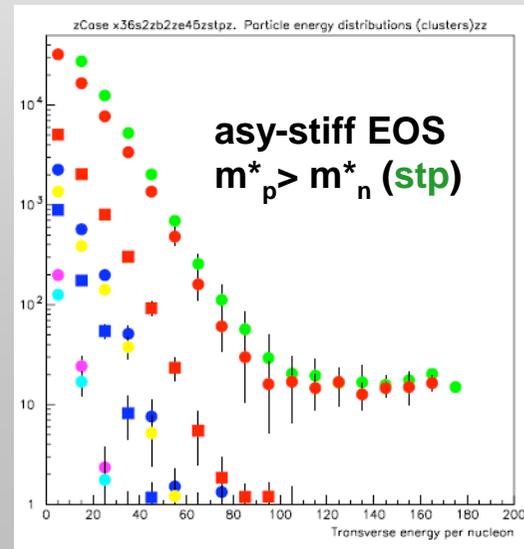
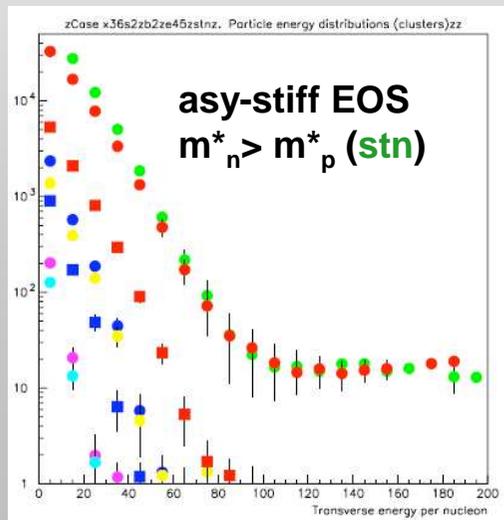
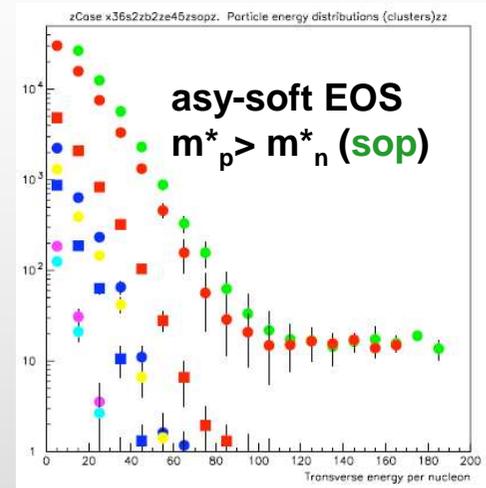
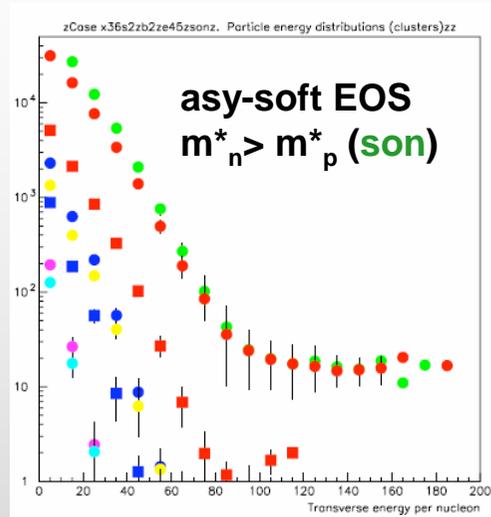
Observable very sensitive at high p_T
to the mass splitting and **not** to the asy-stiffness

V.Giordano, M.Colonna et al., PRC 81(2010)

Crossing of
the symmetry potentials for
a matter at $\rho \approx 1.7 \rho_0$

Emission of light clusters

Particle Yields $^{136}\text{Xe} + ^{124}\text{Sn}$, 32 A MeV

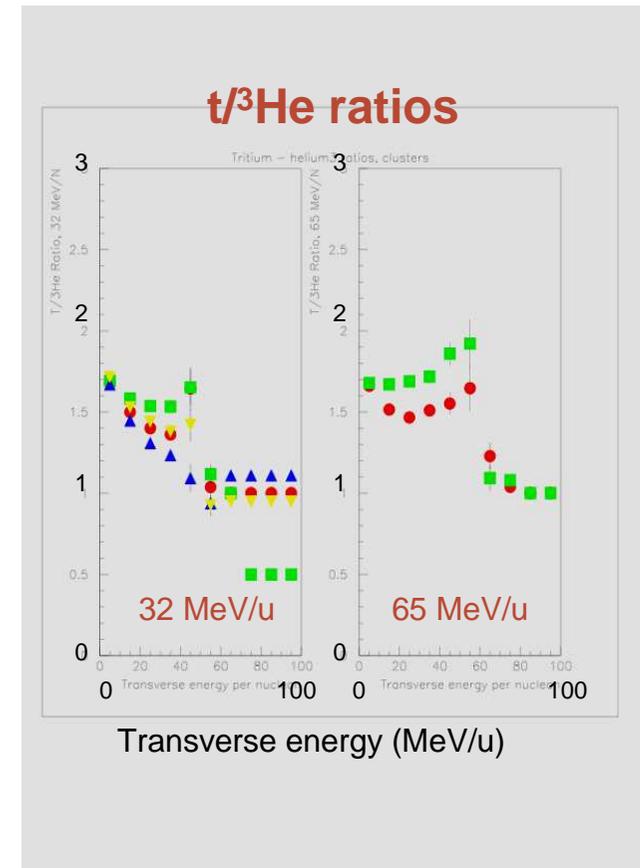
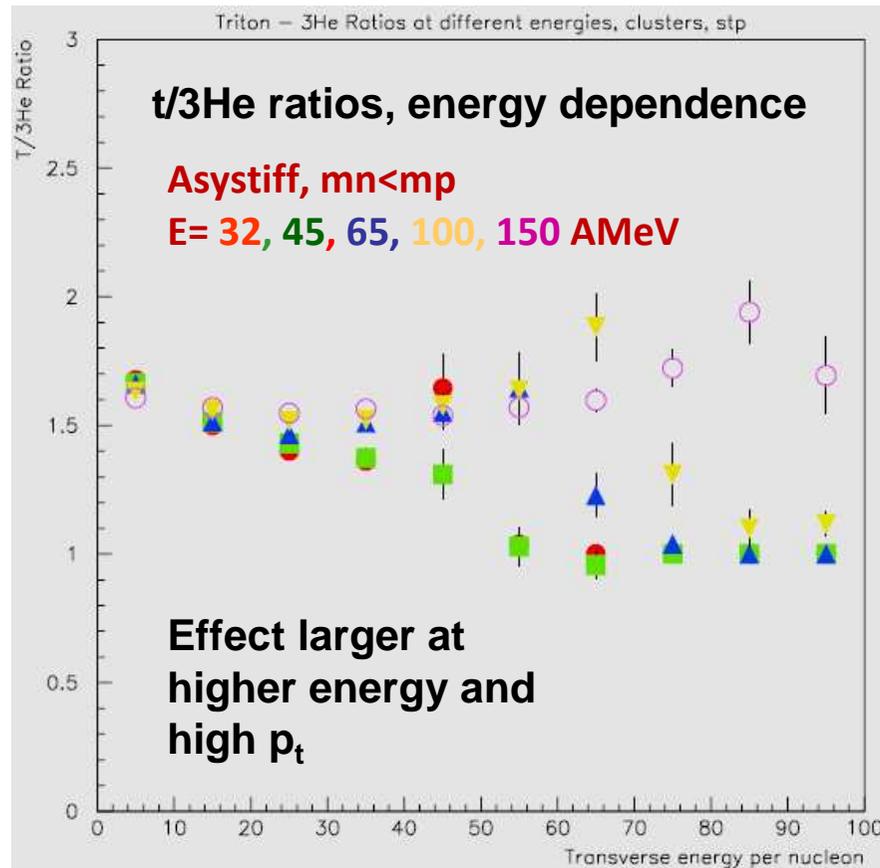


- n
- p
- d
- t
- ^3He
- α
- ^7Li
- ^7Be

n/p and $t/{}^3\text{He}$ ratios as functions of transverse energy

136Xe+124Sn

M. Zielinska-Pfabe, IWM09



- green** asy-soft EOS $m_p^* > m_n^*$ (**sop**)
- red** asy-stiff EOS $m_p^* > m_n^*$ (**stp**)
- blue** asy-stiff EOS with $m_n^* > m_p^*$ (**stn**)
- yellow** asy-soft EOS with $m_n^* > m_p^*$ (**son**)

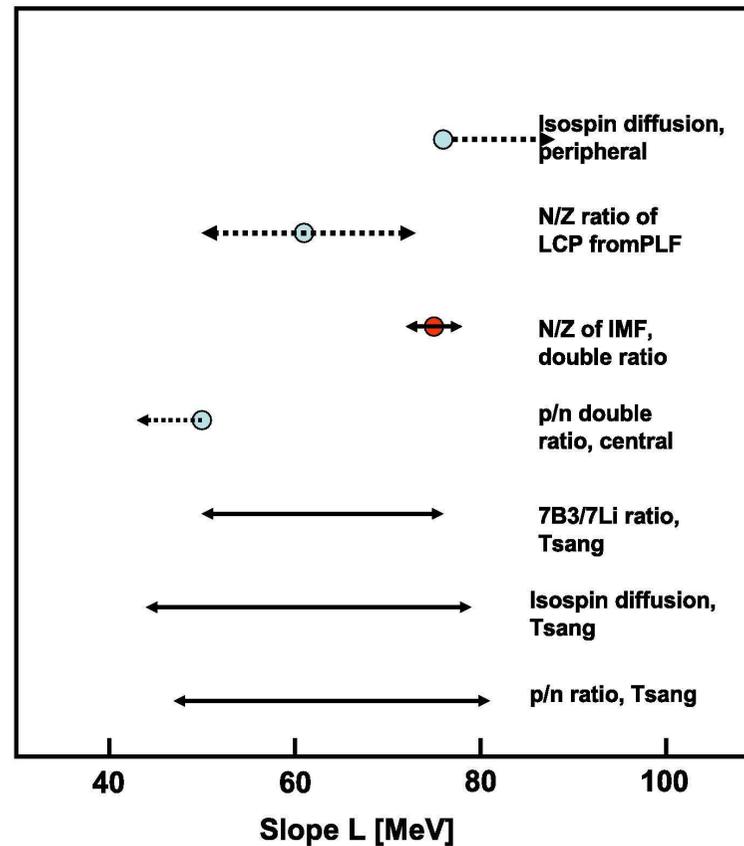
sop and **stp** n accelerate more than p, the ratio goes up. (There is a limit for first chance emission (beam+ Fermi)).

stn and **son** decrease only. For $t/{}^3\text{He}$ goes down about beam energy (prob. statistics).

More sensitivity to effective mass than to symmetry energy!

Slope of Symmetry Energy

Comparison of results for symmetry energy slope
extracted from Fermi energy collisions



Catania results,
qualitative
 $S_0=30$ MeV

M.B. Tsang,
et al.,
PRL102,1227
01(09)

Agreement in general but differences in detail.

Point to different dynamics in the different approaches which should be better understood, as e.g. in Colonna et al., PRC 81 (2011)

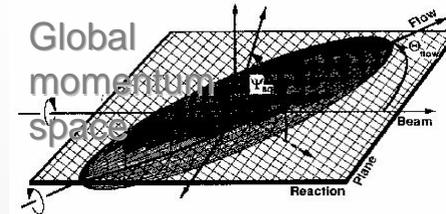
Heavy Ion Collisions at Relativistic Energies: "Flow"

Fourier analysis of momentum tensor : „flow“

$$N(\theta, y, b) = N_0(1 + v_1(y, b)\cos\theta + v_2(y, b)\cos 2\theta + ..)$$

v_1 : directed flow

v_2 : elliptic flow



To investigate symmetry energy:

differences of flow (more sensitive for clusters):

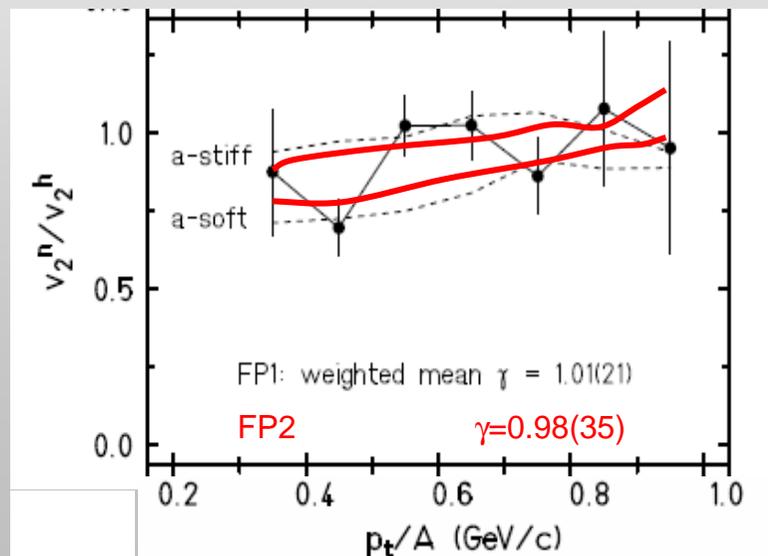
or differential flow

(analogous for $v_1, v_2, ..$)

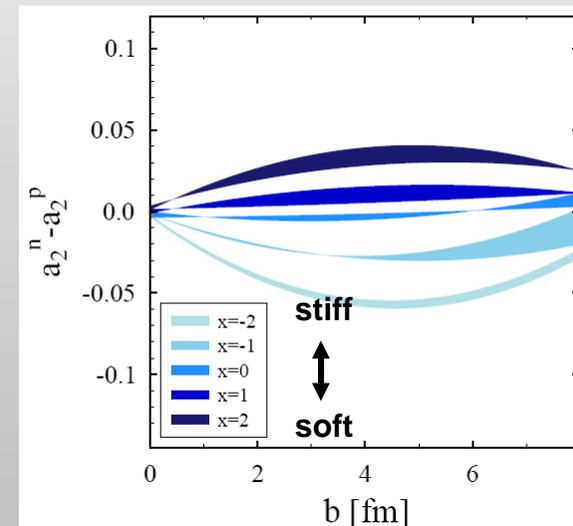
$$\langle p_x^t / A \rangle - \langle p_x^{3\text{He}} / A \rangle = \frac{1}{N_t} \sum_{i=1}^{N_t} p_x^i / A - \frac{1}{N_{3\text{He}}} \sum_{i=1}^{N_{3\text{He}}} p_x^i / A.$$

$$\langle p_x^{t-3\text{He}} / A \rangle = \frac{N_t}{N_t + N_{3\text{He}}} \langle p_x^t / A \rangle - \frac{N_{3\text{He}}}{N_t + N_{3\text{He}}} \langle p_x^{3\text{He}} / A \rangle,$$

Au+Au @ 400 AMeV, FOPI-LAND



Russotto, et al., PLB 697, 471 (2011)



Recent study by Cozma, arxiv 1102.2728

band: soft vs. stiff eos of **symmetric** matter, \rightarrow robust probe

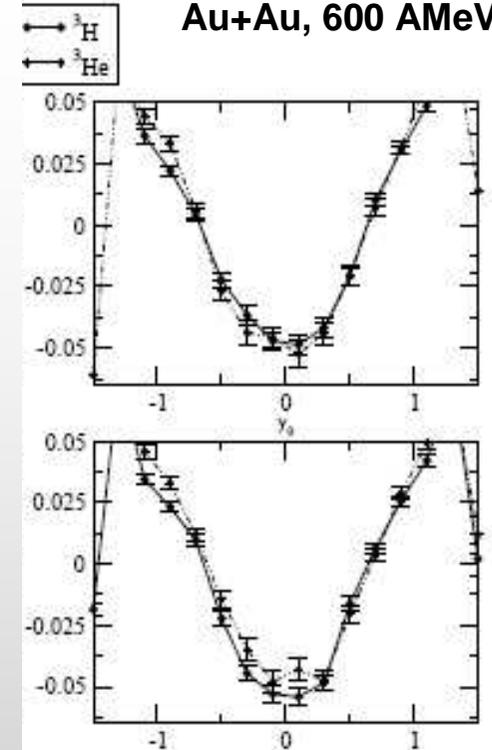
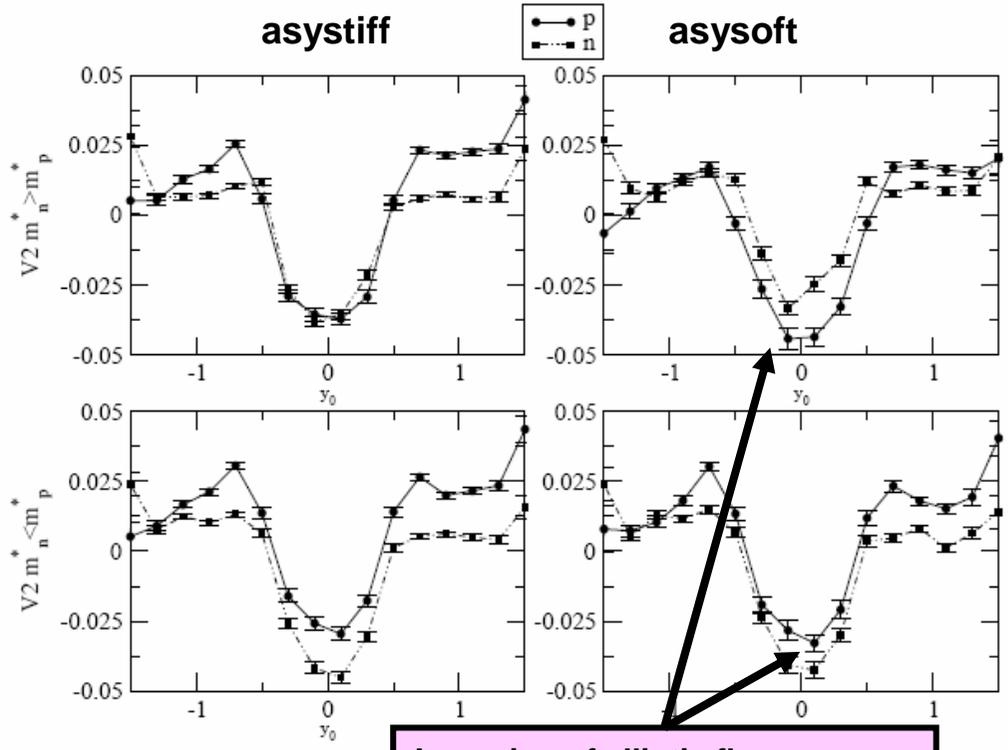
Differential elliptic flow

Au+Au, 400 AMeV

Au+Au, 600 AMeV

$m_n^* > m_p^*$

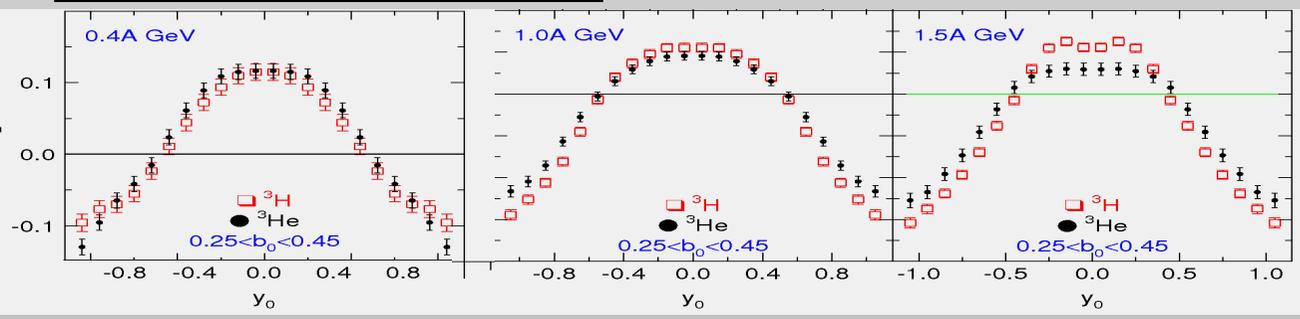
$m_n^* < m_p^*$



Inversion of elliptic flows because of inversion of potentials with effective mass

t - ^3He pair similar but weaker

W. Reisdorf, ECT*, May 09
Indication of experimental effect

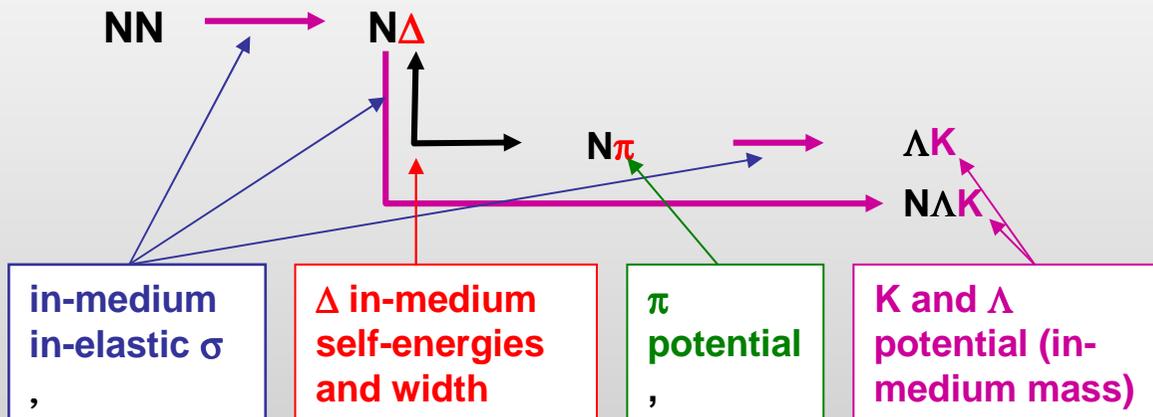
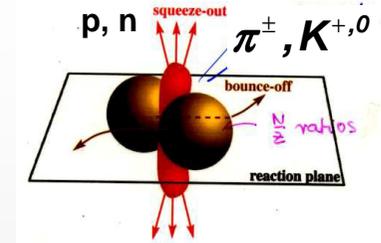


Elliptic flow more sensitive, since determined by particles that are emitted perp to the beam direction

Particle production as probe of symmetry energy

Difference in neutron and proton potentials

1. „direct effects“: difference in proton and neutron (or light cluster) emission and momentum distribution
2. „secondary effects“: production of particles, isospin partners $\pi^{\pm}, K^{0,+}$



1. Mean field effect: U_{sym} more repulsive for neutrons, and more for asy stiff
 → pre-equilibrium emission of neutron, reduction of asymmetry of residue

$$\frac{n}{p} \downarrow \Rightarrow \frac{Y(\Delta^{0,-})}{Y(\Delta^{+,++})} \downarrow \Rightarrow \frac{\pi^-}{\pi^+} \downarrow$$

decrease with stiffer asyEOS

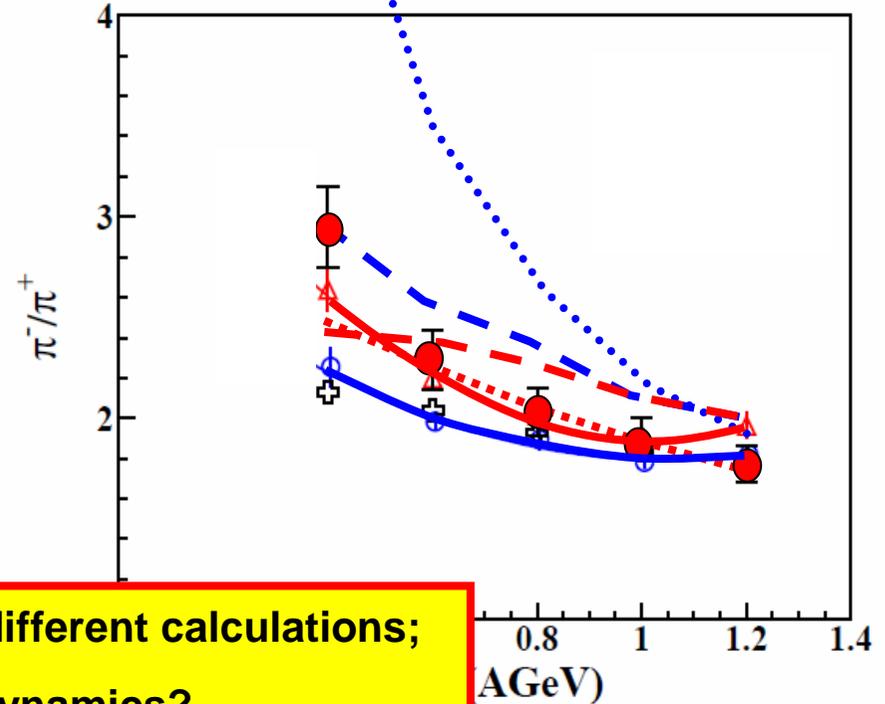
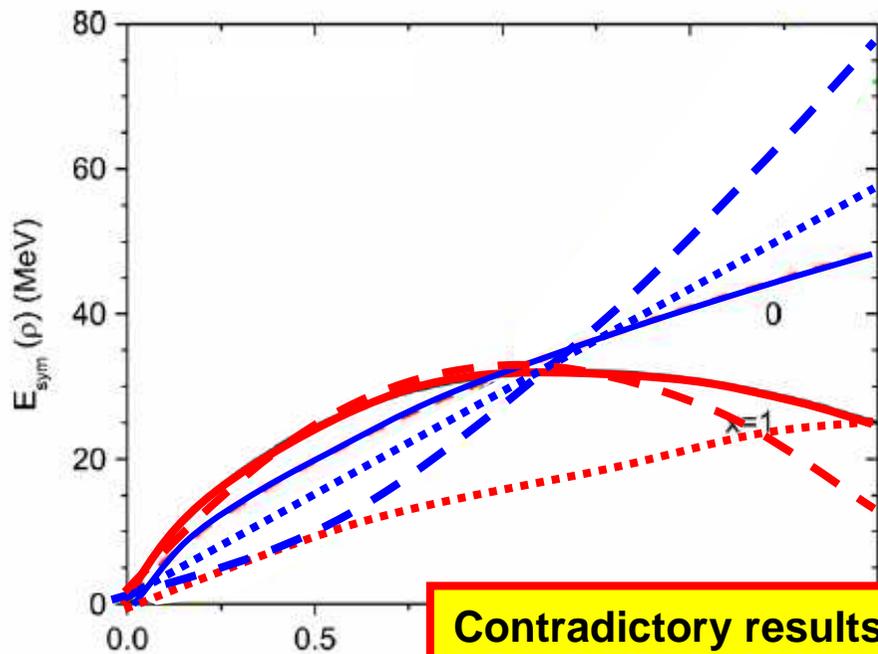
2. Threshold effect, in medium effective masses:

→ m_N^* , m_Δ^* , contribution of symmetry energy; m_K^* , models for K-potentials

$$\sigma = \sigma(s_{in} - s_{th})$$

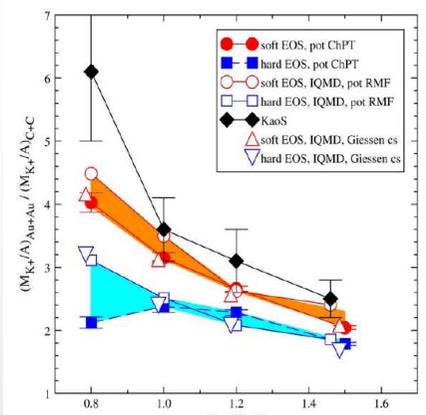
$$\frac{\pi^-}{\pi^+} \uparrow \text{ increase with asy - stiffness}$$

Pion ratios in comparison to FOPI data (W.Reisdorf et al. NPA781 (2007) 459)



**Contradictory results of different calculations;
Reason: Treatment of Δ dynamics?**

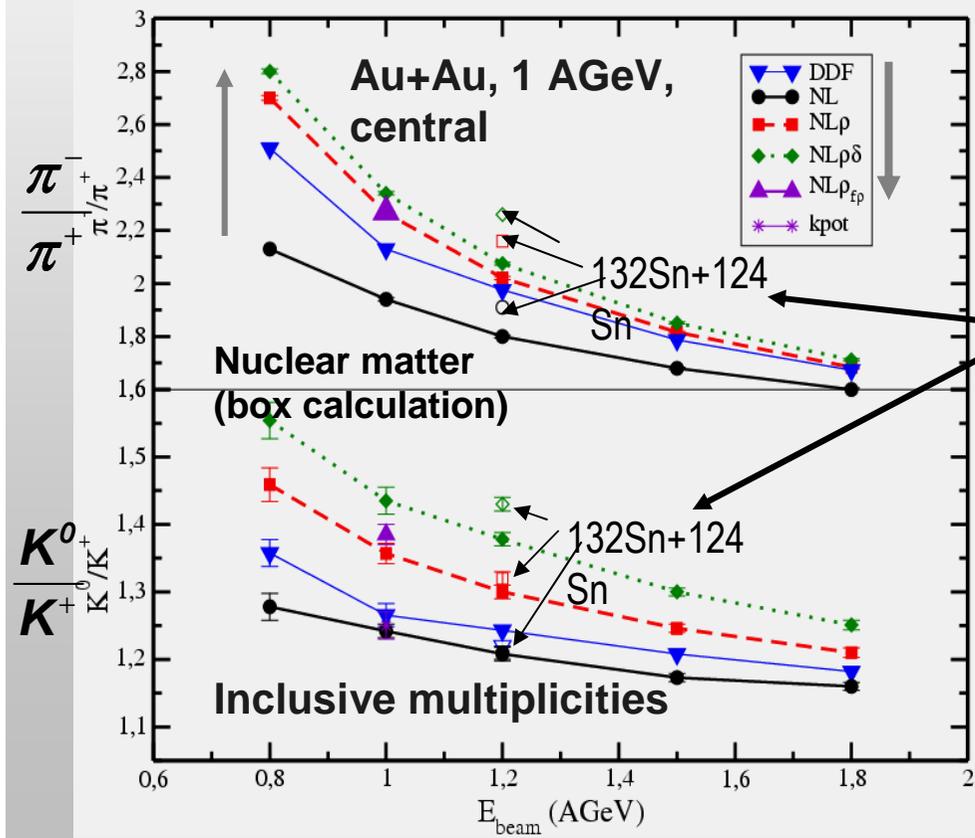
Kaons a good probe for the EOS – also for the Symmetry Energy?



Kaons are a more sensitive and clean probe of the high density EOS. Demonstrated by Fuchs, et al., PRL 86 (01), C.M. Ko & J. Aichelin, PRL55(85) for **symmetric** matter in comparison with KAOS data.

Kaons are **closer to threshold**, come only **from high density**, have **large mean free path**, **small width**:

Also a probe for the symmetry energy for ratio of different charge states, K^0/K^+ ?



From soft to stiff from lower to upper curves:
Stiffer asy-EOS → larger ratio!
 Threshold dominates mean field effect; larger at lower energies

Effect reduced in finite nuclei (pre-equil emission reduces asymmetry)

Kaon ratio still a bit more sensitive probe:
 ~15% difference betw. very soft and stiff
 → small but perhaps measurable!
 → double ratio data by FOPI (Lopez, et al., PRC75 (07))

Summary and Outlook:

- Investigated different probes for the Symmetry Energy in heavy ion collisions with transport theory at different energy (density) ranges
- Explore the sensitivity to the ingredients (asy-EOS, momentum dependence of SymEn, medium, esp. isospin dependence of cross sections)
- Still considerable differences between different codes. Continue code comparison initiative (?)
- Consistent description of many observables mandatory, also from structure and astrophysics
- Many knotty problems!



Trento, Duomo

Thank you!