Nuclear equation of state from neutron stars and core-collapse supernovae

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International Symposium on Nuclear Symmetry Energy (NuSYM11)
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17-20 June, 2011
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Core-collapse supernovae

- Gravitational core-collapse of a star with $M > 8M_\odot$
- Inner core rebounds at $n_b \sim n_0$ ⇒ shock wave formation
- Shock wave crosses neutrinospheres ⇒ burst of neutrinos
- Hot and dense proto neutron star is left after explosion

**Problem:** Shock looses too much energy and stalls as standing accretion shock (SAS) at $r \sim 100$ km

To revive a shock wave

**Neutrino driven** (H.A.Bethe, J.R.Wilson, 1985):
- Neutrinos revive stalled shock by energy deposition, standing accretion shock instabilities (SASI)

**Acoustic mechanism** (A.Burrows et al., 2006):
- G-modes of the core: sound waves steepen into shock waves

**By magnetohydrodynamics** (G.S.Bisnovatyi-Kogan, 1971):
- Transfer of angular momentum via a strong magnetic field

**Phase transition** (I.A.Gentile et al., 1993):
- Collapse of proto neutron star to a more compact hybrid star configuration

Neutron stars

- **Radius:** $R \sim 10\text{km}$
- **Mass:** $M \sim (1 - 3)\,M_\odot$

- **$1.18^{+0.03}_{-0.02}\,M_\odot$:** J1756-2251
  (Faulkner et al., ApJ 618 (2005))

- **$1.4414\pm0.0002\,M_\odot$:** B1913+16

- **$M=1.667\pm0.021\,M_\odot$:** J1903+0327
  (Freire et al., MNRAS, 2011)

- **$M=1.97\pm0.04\,M_\odot$:** J1614-2230
  (Demorest et al., Nature 467, 2010)

Ferdman, R. D., Ph.D thesis (2008): **$1.258^{+0.018}_{-0.017}\,M_\odot$** for J1756-2251
Neutron star interior

Bulk nuclear matter in mechanical, thermal, and weak equilibrium:

- Low temperatures: $T \sim (10^6 - 10^8) \, \text{K}$
- Large central densities: $n_b \gg n_0$
  $(5 \, n_0, 10 \, n_0, \ldots)$
- Large isospin asymmetry: $Y_p \ll 0.5$

Properties of neutron stars (mass, radius) are governed by general relativity and the nuclear equation of state

Figure: F. Weber
Nuclear equations of state

**Phenomenological:**

\[ U(n_b) = \frac{A}{2} \left( \frac{n_b}{n_0} \right) + \frac{B}{\sigma + 1} \left( \frac{n_b}{n_0} \right)^\sigma \]

\[ E_{\text{sym}}(n_b) \sim S_0 \left( \frac{n_b}{n_0} \right)^\alpha \]

- \( BE = -16 \text{ MeV}, n_0 \sim 0.16 \text{ fm}^{-3} \)
- \( K_0 = (160 - 240) \text{ MeV} \)
- \( S_0 = (28 - 32) \text{ MeV}, \alpha = 0.7 - 1.1 \)

**Skyrme and rel. mean field:**

<table>
<thead>
<tr>
<th>Model</th>
<th>( S_0 ) [MeV]</th>
<th>( K_0 ) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bsk8 (NPA 750 (2005))</td>
<td>28.0</td>
<td>230.2</td>
</tr>
<tr>
<td>Sly4 (NPA 635 (1998))</td>
<td>32.0</td>
<td>229.9</td>
</tr>
<tr>
<td>TM1 (NPA 579 (1994))</td>
<td>36.9</td>
<td>281</td>
</tr>
</tbody>
</table>
Masses and radii of hadronic stars

Solving the Tolmann-Oppenheimer Volkov equations with nuclear EoS as input

- Maximum neutron star mass is dependent on compressibility
- Radius of low mass stars depends on symmetry energy
Solving the Tolmann-Oppenheimer Volkov equations with nuclear EoS as input

High central densities $\gtrsim 5n_0$ in maximum mass neutron stars

Onset of quark matter/hyperons (?)
Hyperons in neutron stars

- Microscopic calculations: Inclusion of hyperons softens the EoS
- For stiffer EoS Hyperons appear at lower densities (H.-J. Schulze et al. PRC 73, 2006)
- Hyperons populate neutron stars and lead to (too) low maximum masses

- Quark meson-coupling model (QMC), J. Rikovska Stone et al., NPA, Vol 792, 2007
- SU(3) non-linear sigma model, V. Dexheimer and Schramm, PRC, Vol. 18, 2010
- RMF (TM1) extended to Λ, Σ, Ξ, Bednarek and Manka, J. Phys. G, 36 (2009) with interaction up to quartic terms in fields

Hybrid stars with the Bag model

- Quark EoS: Bag model
  
  \[ p(\mu_i, T) = \sum_i p_F(\mu_i, T) - B \]
  
  \[ \epsilon(\mu_i, T) = \sum_i \epsilon_F(\mu_i, T) + B \]

  \( i = \) up, down, strange

- Phase transition influenced by:
  - Local (Maxwell) or global (Gibbs) charge conservation
  - Number of degrees of freedom (e.g. strangeness)
  - Proton fraction / Symmetry
  - Energy of hadronic matter
  - Temperature
Hybrid stars with the Bag model

- First order corrections in strong interaction coupling constant (Farhi & Jaffe, PRD30, 1984)

\[ p_f(\mu_f, a_4) = p_f(\mu_f) - \frac{\mu_f^4}{4\pi^3}(1 - a_4) \]
Various quark matter models fullfill a two-solar mass constrain

See also talk by Veronica Dexheimer!

What we have seen so far ...

- A $\sim 2M_\odot$ star restricts the nuclear EoS to stiff parameter sets

- Hadronic matter:
  - Problem for: $K_0 \lesssim 200\text{MeV}$ and supersoft symmetry energy (?)
  - $2M_\odot$ stars with Skyrme-type EoSs: $\rightarrow$ central densities $\gtrsim 5n_0$
  - Stiffer hadronic equations of state (RMF), onset of hyperons or quarks

- Hyperons:
  - Microscopic calculations: hyperons populate neutron star interior
  - But: Nuclear EoS becomes too soft $\rightarrow$ neutron star masses are too low
  - Missing hyperon physics at higher densities ?
  - High mass neutron stars & hyperons in e.g. quark-meson coupling model, SU(3) nonlinear sigma model, extended RMF model

- Quarks:
  - Effects from strong interaction (and color superconductivity) stiffen the quark matter EoS
  - Presence of quark matter and/or a low critical transition density are not excluded
Supernova simulations

- General relativistic hydrodynamics in multi-D
- Neutrino transport
- Weak interaction reaction rates (for electron and neutrino capture)

- Nuclear matter EoS:
  \( T : (0 - \geq 100) \) MeV
  \( Y_p : 0.01 - \geq 0.5 \)
  \( n_b : (10^5 - \geq 10^{15}) \frac{g}{cm^3} \)

- Typical SN conditions around core-bounce:
  \( T \sim 15 \) MeV
  \( Y_p \sim 0.3 \)
  \( n_b \sim n_0 \sim 0.16 \) fm\(^{-3}\)

T. Fischer et al., APJS, Vol. 194, (2011)
Hadronic equations of state for supernova simulations

**Lattimer-Swesty (LS) equation of state:**
- Based on Skyrme type interaction with two and multibody term
- \( S_0 = 29.3 \text{MeV}, K_0 = 180, 220, 375 \text{MeV}, n_0 = 0.155 \text{fm}^{-3} \)
- Components: Neutrons, protons, \( \alpha \) particles, and representative heavy nucleus
- Compressible liquid drop model
- Simplified treatment of pasta phases between \( 1/10n_0 \) - \( 1/2n_0 \)

**Shen et al. (Shen) equation of state:**
- Relativistic mean field, TM1 (fitted to Relativistic Brueckner Hartree Fock and properties of neutron rich nuclei)
- \( S_0 = 36.9 \text{MeV}, K_0 = 281 \text{MeV}, n_0 = 0.145 \text{fm}^{-3} \)
- Components: Neutrons, protons, \( \alpha \) particles, representative heavy nucleus
- Thomas-Fermi calculations
- No pasta phases, nuclei are spherical, no shell effects
LS and Shen in Simulations - Shock wave and Neutrinos

1D general relativistic core-collapse supernovae simulation with $\nu$ radiation hydrodynamics

Stiffer Shen EoS leads to:

- Smaller free proton fraction and less neutron rich nuclei during collapse phase
- Higher lepton fraction in inner core
- Inner core has a larger radius and higher enclosed mass
- Smaller contraction of the core $\rightarrow$ lower temperature
- Lower neutrino luminosities and neutrino energies
Black hole formation

Systematic study of failing core-collapse supernovae

- Stiffer EoS: longer black hole formation time and larger grav. mass
- Test of required neutrino heating for explosion in dependence of progenitor’s compactness at bounce

\[ \xi_M = \frac{M/M_\odot}{R(M_{\text{bary}} = M)/1000 \text{ km}} \bigg|_{t=t_{\text{bounce}}} \]

- Explosion is the likely outcome of core collapse for progenitors with \( \xi_{2.5} \lesssim 0.45 \) if the nuclear EOS is similar to the LS180 or LS220 case

Gravitational waves

- Negative gradients in entropy and lepton fraction after shock wave passage → prompt convection
- For $t > 100\text{ms}$: convective overturn inside proto neutron star and SASI
- A more compact neutron star leads to more powerful shock oscillations

Wolff-Hillebrandt: Skyrme Hartree-Fock, $K_0 = 263\text{MeV}, S_0 = 32.9\text{MeV}$
Nuclei are important

Single nucleus approximation should be replaced by an ensemble of nuclei

- During core collapse: electron capture on free protons and nuclei → determines lepton fraction at bounce and size of the core
- Neutrino spectra are formed at the neutrionsphere as where $\rho \sim 10^{11}\text{g/cm}^3$
- Shock stalls at densities of $\rho \sim 10^9\text{g/cm}^3$
- Additional neutrino heating behind the stalled shock front due to neutrino nucleus interaction

Figures taken from Janka et al., Phys. Rep. 442 (2007), correspond to presupernova stage (top) and neutrino trapping phase (bottom)
A statistical model for a complete supernova EoS


- Thermodynamic consistent nuclear statistical equilibrium model
- Relativistic mean-field model for nucleons, excluded volume effects for nuclei
- New aspects: shell effects, distribution of nuclei, all light clusters
- RMF models: TM1, TMA, NL3, FSUGold

<table>
<thead>
<tr>
<th></th>
<th>( J ) [MeV]</th>
<th>( L ) [MeV/fm(^3)]</th>
<th>( K_0 ) [MeV]</th>
<th>( M_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM1</td>
<td>36.95</td>
<td>110.99</td>
<td>282</td>
<td>2.21</td>
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<tr>
<td>TMA</td>
<td>30.66</td>
<td>90.14</td>
<td>318</td>
<td>2.02</td>
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<tr>
<td>FSUGold</td>
<td>32.56</td>
<td>60.44</td>
<td>230</td>
<td>1.74</td>
</tr>
<tr>
<td>NL3</td>
<td>37.39</td>
<td>118.50</td>
<td>271</td>
<td>2.79</td>
</tr>
</tbody>
</table>

Figures: M. Hempel & J. Schaffner-Bielich, NPA, Volume 837; Table: M. Hempel, EoS user manual
RMF and Virial Equation of State for Astrophysical Simulations


- RMF for uniform matter at high $n_b$, Hartree intermed. $n_b$ for non-uniform matter, Virial gas expansion at low density
- Components: Neutrons, protons, $\alpha$ particles, and nuclei from FRDM
- Aspects: shell effects, spherical pasta phases
- RMF parameter set: NL3, FSUGold, FSUGold2.1

(Some) further Models

Nuclear Statistical Equilibrium:

Other Approaches:

Hyperons
- Ch. Ishizuka et al., JPG Vol. 35, 2008; inclusion of $\Lambda$, $\Sigma$, $\Xi$ in Shen EoS
- M. Oertel and A. Fantina, in SF2A-2010, hyperons and thermal pions in LS EoS
- H. Shen et al., arXiv:1105.1666, 2011, extended and refined Shen EoS with inclusion of $\Lambda$ hyperons
Hyperons in supernovae


Sumiyoshi et al., ApJL, Vol. 690, 2009:
- Shen equation of state with hyperons and thermal pions
- 1D general relativistic supernova simulation with neutrino-radiation hydrodynamics
- Collapse of a 40M⊙ progenitor
- Hyperons appear around 500ms after core bounce
- Earlier black hole formation at around 682ms postbounce than for normal Shen EoS

Quark Matter in supernovae - high critical density

Nakazato et al., Phys.Rev.D, 77, 2008:
- Shen EoS with phase transition to quark matter at $\rho \gtrsim 5\rho_0$
- Quark matter with Bag model
- Core collapse SN of a 100 $M_\odot$ progenitor
- Phase transition shortens time until black hole formation

T. Fischer et al., CPOD2010
- Shen EoS with phase transition to quark matter with PNJL model
- Core collapse of a 15$M_\odot$ progenitor
- Late appearance of strange quarks
- No phase transition during post-bounce accretion phase
Quark matter in supernovae - low critical density

- Shen EoS with phase transition to quark matter at $\rho \gtrsim \rho_0$
- GR hydrodynamics and Boltzmann neutrino transport in spherical symmetry (Liebendoerfer et al. 2004)
- Softening of the EoS in mixed phase for higher quark fractions $\rightarrow$ contraction of the PNS till pure quark matter is reached
- Formation of second shock front which turns into shock wave
- Second shock wave accelerates and leads to the explosion of the star

First and Second Neutrino Bursts

- Second shock wave passes neutrinospheres → second neutrino burst dominated by antineutrinos.

- For $B^{1/4} = 165\text{MeV}$ second neutrino burst is $\sim 200$ ms later than for $B^{1/4} = 162\text{MeV}$.

Fig: T.Fischer, Neutrino luminosities and rms neutrino energies, at 500km for $10\, M_\odot$ progenitor.
### Results (Fischer et al., arXiv:1011.3409)

<table>
<thead>
<tr>
<th>Prog. $M_\odot$</th>
<th>$B^{1/4}$ MeV</th>
<th>$t_{pb}$ ms</th>
<th>$\rho_c$ $10^{14}$g/cm$^3$</th>
<th>$T_c$ MeV</th>
<th>$M_{pns}$ $M_\odot$</th>
<th>$E_{expl}$ $10^{51}$erg</th>
<th>$M_{max}$ $M_\odot$</th>
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<tbody>
<tr>
<td>10.8</td>
<td>162</td>
<td>240</td>
<td>6.61</td>
<td>13.14</td>
<td>1.431</td>
<td>0.373</td>
<td>1.55</td>
</tr>
<tr>
<td>10.8</td>
<td>165</td>
<td>428</td>
<td>6.46</td>
<td>14.82</td>
<td>1.479</td>
<td>1.194</td>
<td>1.50</td>
</tr>
<tr>
<td>13</td>
<td>162</td>
<td>235</td>
<td>6.49</td>
<td>13.32</td>
<td>1.465</td>
<td>0.232</td>
<td>1.55</td>
</tr>
<tr>
<td>13</td>
<td>165</td>
<td>362</td>
<td>7.23</td>
<td>16.38</td>
<td>1.496</td>
<td>0.635</td>
<td>1.50</td>
</tr>
<tr>
<td>15</td>
<td>162</td>
<td>209</td>
<td>7.52</td>
<td>17.15</td>
<td>1.608</td>
<td>0.420</td>
<td>1.55</td>
</tr>
<tr>
<td>15</td>
<td>165</td>
<td>276</td>
<td>7.59</td>
<td>16.25</td>
<td>1.641</td>
<td>u</td>
<td>1.50</td>
</tr>
<tr>
<td>15, $\alpha_s = 0.3$</td>
<td>155</td>
<td>326</td>
<td>5.51</td>
<td>17.67</td>
<td>1.674</td>
<td>0.458</td>
<td>1.67</td>
</tr>
</tbody>
</table>

- **Larger Bag ($B^{1/4} = 165\text{MeV}$):**
  - Higher critical density $\rightarrow$ Longer accretion on proto neutron star
  - More massive proto neutron star with deeper gravitational potential
  - Stronger second shock and larger explosion energies
  - Second neutrino burst later with larger peak luminosities

- More massive progenitor: earlier onset of phase transition and more massive proto neutron star
Summary

- Robust explosion mechanism/trigger for explosion is still missing
- For a long time - only two mainly used nuclear equations of state
- Neutrino signal, black hole formation time, and gravitational wave signal are sensitive to the nuclear equation of state
- But: No systematic study on the influence of the symmetry energy
- Hooray: New equations of state are on their way! Different parameter sets, distribution of light and heavy clusters, pasta phases, ... and exotica
- Up to now: Inclusion of hyperons shows no significant influence on supernova/proto neutron star dynamics
- Low density quark matter phase transition in the early post-bounce phase of core-collapse supernova can trigger the explosion and can be verified by the observation of a second neutrino burst
- Phase transition induced supernova has to be tested for compatibility with a two solar mass star