Clusters in low density matter and the EoS

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Outline

- Motivation
- Composition of nuclear matter. Constraints
- Low densities. Theoretical approaches
- Symmetry energy T=0
- Quantum statistical approach
- Generalized RMF model with light clusters
- Symmetry energy for finite T
- Generalized RMF model with continuum correlations
- Results and outlook

Motivation

- Description of various phenomena in nuclear physics requires knowledge of the equation of state
- Many EoS, which describe different density and temperature regions: from simple parametrizations to sophisticated models

- These models are often restricted to particular conditions:
  - symmetric nuclear matter
  - neutron matter
  - low/high densities
  - matter in β-equilibrium

Aim:

Construct a single phenomenological model, covering full parameter space (in densities, temperatures and proton asymmetries)

→ We need a combination of different approaches

EoS used in astrophysical models

- J.M. Lattimer, F.D. Swesty, Nucl. Phys. A 535 (1991) 331

simple parametrization for energy density of matter and heavy nuclei, only α clusters, parameters from calculations with Skyrme forces.

- H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi, Prog. Theor. Phys. 100 (1998) 1013

non-linear relativistic mean-field model, Thomas-Fermi calculation, particles and clusters as in LS

More recent EoS for astrophysical applications:

- M.Hempel, J. Schaffner-Bielich (Nucl. Phys. A 837 (2010) 210)
- A.S Botvina, I. N. Mishustin (Nucl. Phys. A 843 (2010) 98)
- G. Shen, C. J. Horowitz, S. Teige (Phys. Rev. C (2010) 015806)
- G. Shen, C. J. Horowitz, E. O’Connor (arXiv:1103.5174)

→ Large variety of possible particles and different treatment of interactions between them

Composition of nuclear matter

- Depends strongly on density, temperature and n-p asymmetry
- Affects thermodynamical properties

- Low densities: mixture of nuclei and nucleons,
  models with nuclei in statistical equilibrium
  (NSE, virial expansion, Beth-Uhlenbeck,…)
- High densities (around/above ρ_{sat}) :
  homogeneous and isotropic neutron-proton matter,
  mean-field models
  (Skyrme, Hartree-Fock, relativistic mean-field,…)
- In between at low temperatures: "liquid-gas" phase transition
  surface effects and long-range Coulomb interaction, inhomogeneous matter, formation of "pasta" phases/lattice structures

→ Interpolation between low-density and high-density limit needed

Here consider: quantum statistical approach, generalized RMF model

Constraints

Constraints for EoS models:

- **Nuclear physics:**

- Properties of nuclei (binding energies, surface properties)
- Nuclear matter parameters (binding energy per nucleon, ρ_{sat} , J, K, K', L)
- Heavy ion collisions (elliptic flow)

- **Astronomy**

- Maximum mass of Neutron star
- Cooling history of NS

Remark:

Most constraints for zero/low temperatures

<u>Development of an improved EoS:</u>

→

- better constrained parameters at low and high densities
- additional particles (clusters at low ρ, hyperons, … at high ρ)
- widest possible range of n, T, Y_p

Low densities. Theoretical models

- NSE:

- Ideal mixture of nucleons and nuclei in chemical equilibrium
- Interaction between particles not considered, no medium effects
- no dissolution of nuclei at high densities

- Virial equation of state in classical description:

- Expand grand-canonical partition function Q in powers of $z_i = e^{\frac{\mu_i}{T}} << 1$ with μ_i non-relativistic chemical potential
- Maxwell-Boltzmann statistics
- Two-body correlations encoded in second virial coefficient b_{ij}
- Particle properties not affected by medium
- No dissolution of nuclei at high densities
- Limitation:

$$\rho\lambda_i^3 << 1, \;\; \lambda_i = \left(\frac{2\pi}{m_i T}\right)^{\frac{1}{2}}$$

Low densities. Theoretical models

- Virial equation of state in quantum mechanical generalization

(G. E. Beth and E. Uhlenbeck Physica 3 (1936) 729, Physica 4 (1937) 915)

- Second virial coefficient with two-body density of states:

$$b_{ij} = \frac{1+\delta_{ij}}{2} \frac{\lambda_i^{\frac{3}{2}} \lambda_j^{\frac{3}{2}}}{\Lambda_{ij}^3} \int dE \; D_{ij}(E) e^{\left(-\frac{E}{T}\right)} \pm g_i 2^{-\frac{3}{2}}$$

$$D_{ij}(E) = \sum_k g_k^{ij} \delta\left(E - E_k^{ij}\right) + \sum_l g_l^{ij} \frac{1}{\pi} \frac{d\delta_l^{ij}}{dE}$$

Contributions from bound states with energies E_k^{ij}

Contributions from continuum states with phase shifts $\delta_l^{ij}(E)$

- Corrections from Bose-Einstein or Fermi-Dirac statistics $\pm g_i 2^{-\frac{3}{2}}$
- with experimental bound state energies/phase shifts

→ Low-density behavior of EoS is established model-independently

(e.g. C.J. Horowitz, A. Schwenk, Nucl. Phys. A776, 55 (2006).)

Symmetry Energy

- General definition for zero temperature

$$E_s(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n,\beta)\Big|_{\beta=0}, \quad \beta = \frac{n_n - n_p}{n_n - n_p}$$

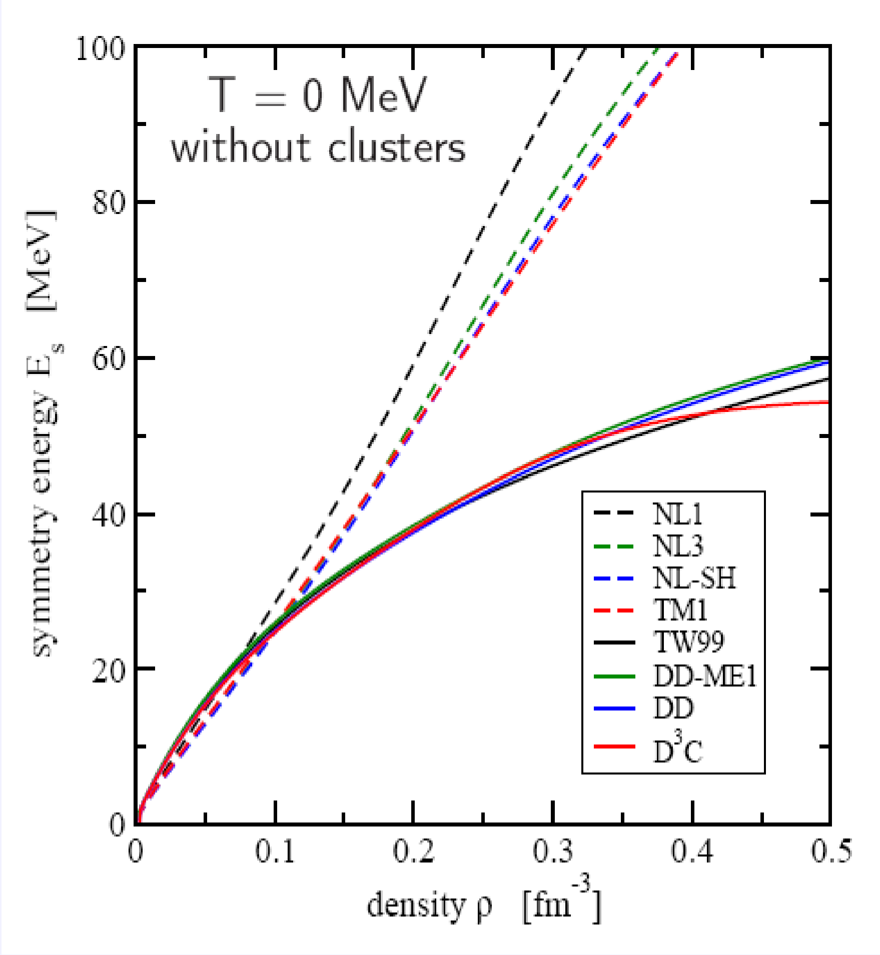
nuclear matter parameters

$$J = E_s(n_{sat}), \; L = 3n \frac{d}{dn} E_s \Big|_{n=n_{sat}}$$

- Neutron skin thickness → slope of neutron matter EoS

(B. A. Brown, Phys. Rev. Lett. 85 (2000) 5296)

(S. Typel, B. A. Brown, Phys. Rev. C 64 (2001) 027302



- Isospin dependence of interaction:
- traditional RMF models with non-linear meson self interactions
- RMF models with density-dependent meson-nucleon couplings (very flexible, well suited for extrapolation)

→ Fit of binding energies and surface properties (independent quantities)

Symmetry Energy

- Temperature T=0 MeV

- Mean-field models without clusters:
  MDI-model with momentum dependent interaction, parameter x controls density dependence of E_{sym}
  (B. A. Li et al., Phys. Rep. 464 (2008) 113)

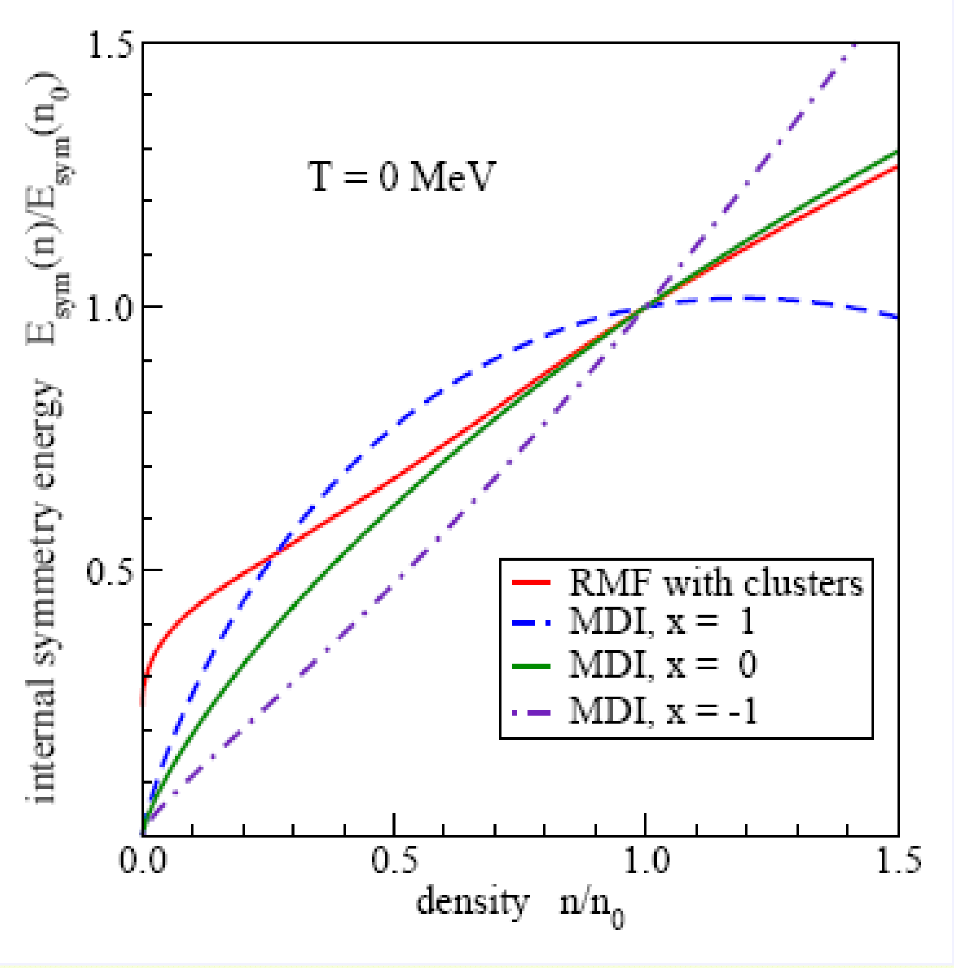
→ low-density behavior not correct

- RMF model with heavy clusters:

→ increase of E_{sym} at low density due to formation of clusters

↓

Finite symmetry energy in the limit n→0



Quantum statistical approach

- Nonrelativistic finite-temperature Green's function formalism
- Starting point: nucleon number densities: (τ=p,n)

$$n_\tau(T,\tilde{\mu}_n,\tilde{\mu}_p) = 2\int \frac{d^3k}{(2\pi)^3} \int \frac{d\omega}{2\pi} f_\tau(\omega) S_\tau(\omega)$$

and spectral function $S_\tau(\omega)$ depending on self-energy Σ_τ

- Expansion of spectral function beyond quasiparticle approximation
- ⟶ Generalized Beth-Uhlenbeck description with

- Medium dependent self-energy shifts/binding energies
- Generalized scattering phase shifts from in-medium T-matrix

$T, n_n, n_p \rightarrow \tilde{\mu}_n, \tilde{\mu}_p \rightarrow F(T, n_n, n_p)$ free energy from integration $\left.\frac{\partial(F/V)}{\partial n_\tau}\right|_{T,n_\tau} = \tilde{\mu}_\tau$

⟶ Thermodynamically consistent derivation of EoS

Quantum statistical approach

Medium modifications

- Single nucleon properties

- self-energy shift of quasiparticle energy
- Effective mass

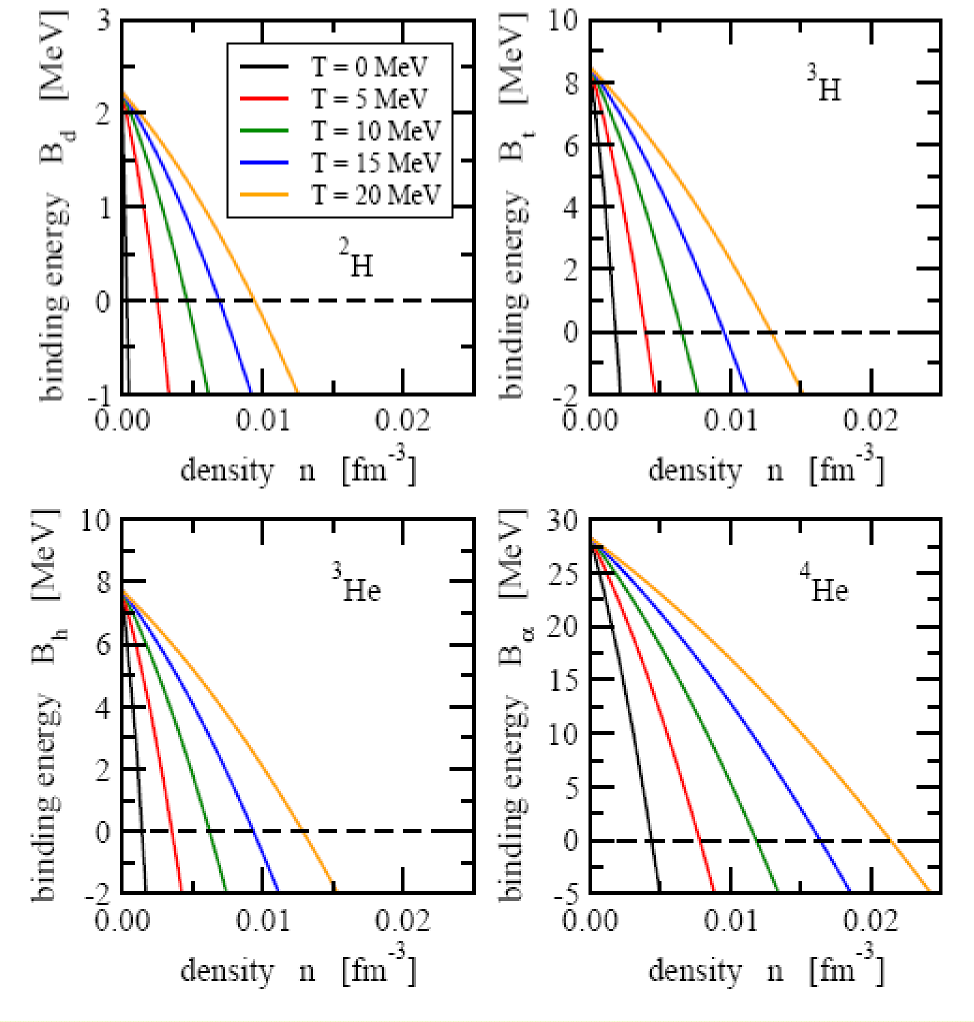
- cluster properties

- Shift of quasiparticle energy from nucleon self-energies

  Pauli blocking

→ medium dependent binding energies (calculation with effective nucleon-nucleon potential)

Symmetric nuclear matter



Generalized RMF model

- Extended relativistic Lagrangian density of Walecka type with nucleons (ψ_p, ψ_n), deuterons (φ_d^μ), tritons (ψ_t), helions (ψ_h), α-particles (φ_α), mesons ($\sigma, \omega_\mu, \vec{\rho}_\mu$), electrons ($\psi_e$) and photons ($A_\mu$) as degrees of freedom

- density-dependent meson-nucleon couplings

$$\Gamma_i(\rho) = \Gamma_i(\rho_{ref}) \cdot f_i\left(\frac{\rho}{\rho_{ref}}\right) \qquad \rho_{ref} \approx \rho_{sat} \approx 0.15\ fm^{-3}$$

- Parameters, constrained from fit to properties of finite nuclei: nucleon/meson masses, coupling strengths (density dependence)

- Medium-dependent cluster binding energies

→ nucleon/cluster/meson/photon field equations, solved self-consistently in mean-field approximation

Generalized RMF model-Constraints

- from finite nuclei
  (present parametrization)
  - binding energies, spin-orbit splittings
  - charge/diffraction radii
  - surface/neutron skin thickness

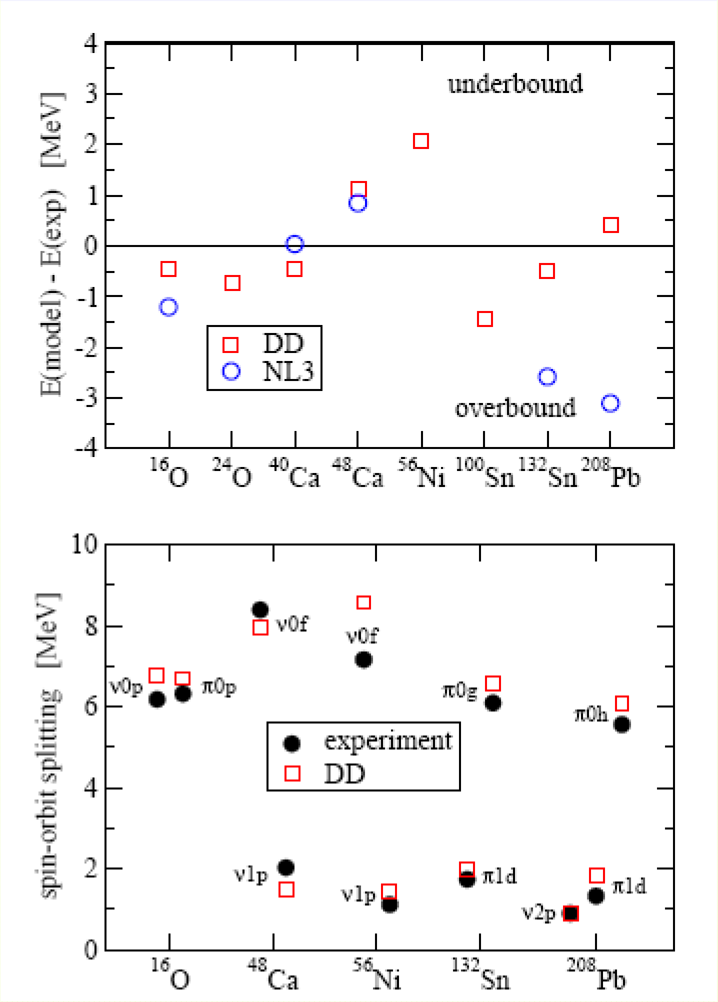
⇒ couplings near ρ_{sat}

⇒ nuclear matter parameters:

ρ_{sat} , E/A ,K , J , L

- from nucleon-nucleon scattering
  - low-energy s-wave phase shifts
  - scattering lengths
  - virial coefficients

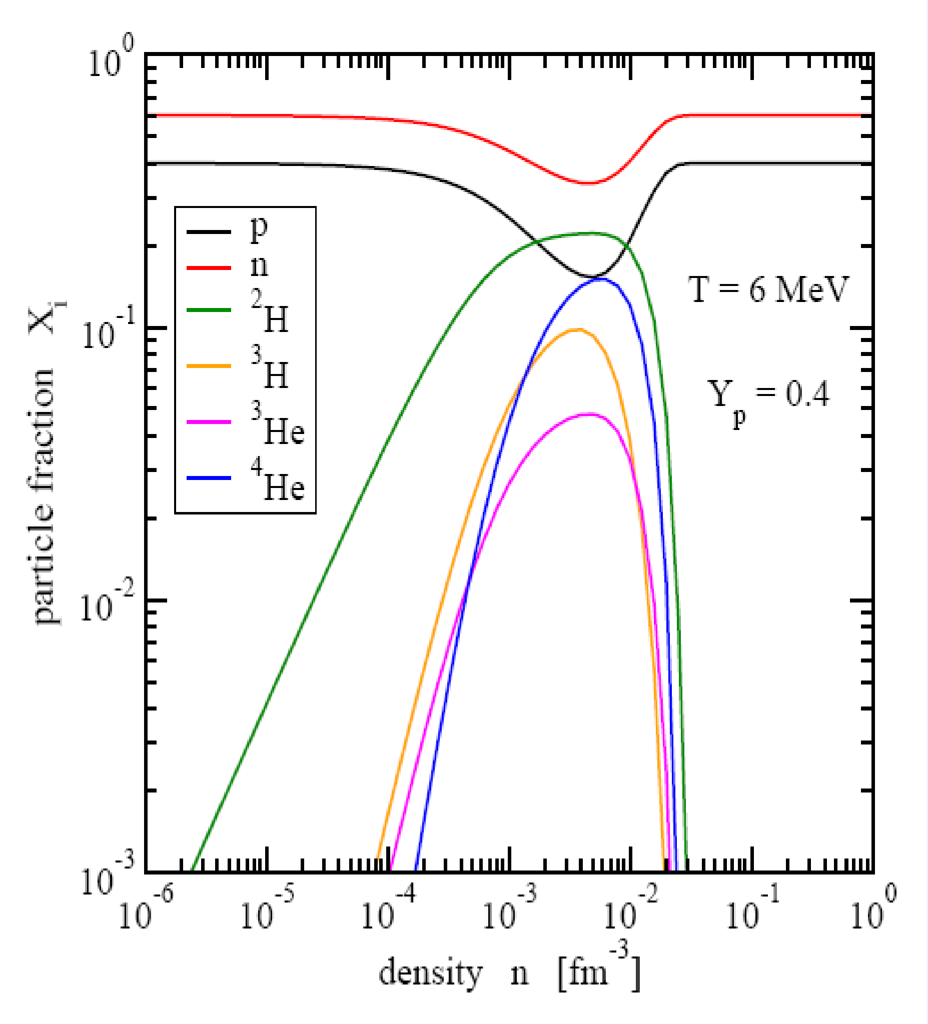
⇒ couplings at zero density



NL3: G.A. Lalazissis, J. König, P. Ring, Phys. Rev. C 55 (1997) 540

EoS with light clusters

- Consider 2-, 3-, 4- body correlations in the medium
- Only bound states presently(d,t,he,α)
- No scattering correlations
- Mott effect: clusters dissolve at high ρ
- correct limits at low and high ρ
- no heavy clusters/phase transition
- medium dependence of couplings and binding energies



→ “rearrangement” contributions in self-energies and source densities (essential for thermodynamical consistency)

Symmetry energy with light clusters

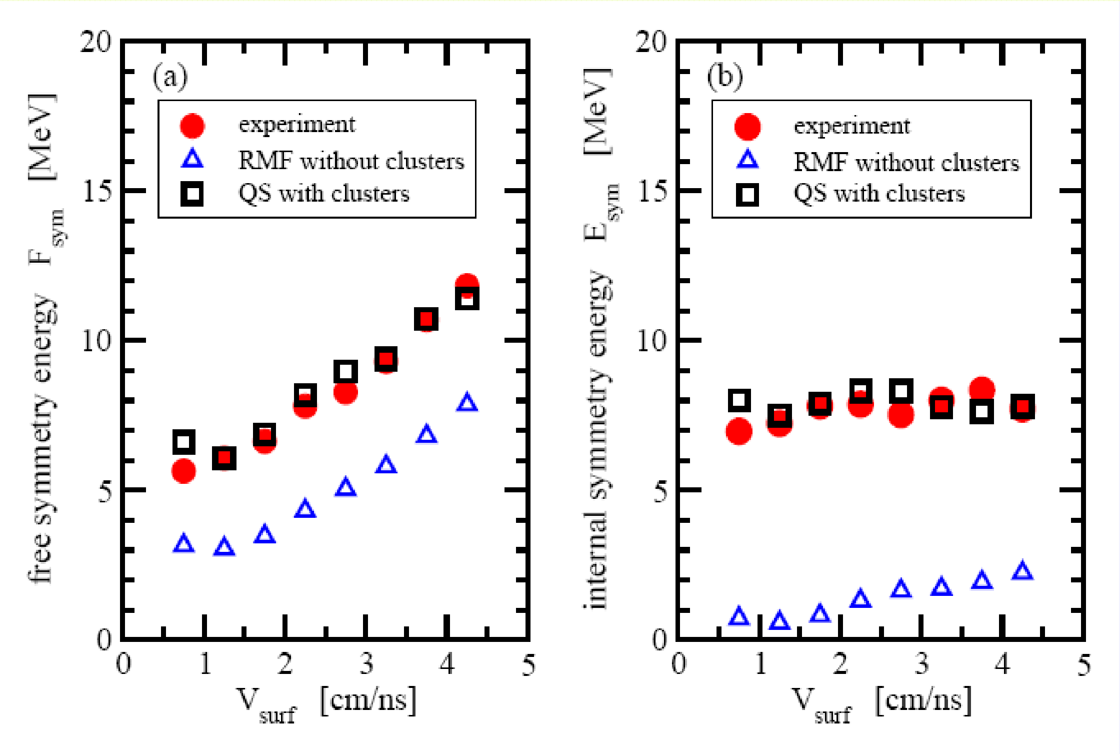
- Finite temperature

- Experimental determination of symmetry energy

heavy-ion collisions of ^{64}Zn on ^{92}Mo and ^{197}Au at 35 A MeV temperature, density, free symmetry energy derived as functions of parameter v_{surf} (measures time when particles leave the source)

(S. Kowalski et al., Phys. Rev. C 75 (2007) 014601)

- symmetry energies in RMF calculation without clusters are too small
- very good agreement with QS calculation with light clusters



Constraints from the virial expansion

Consider generalized RMF description with neutrons, protons. We also incorporate not only the deuteron bound state but also 2-body scattering correlations in the generalized RMF model (in a similar way as deuterons)_as new degrees of freedom.

The density of neutrons and protons:

$$n_i = g_i \int \frac{d^3k}{(2\pi)^3} \left\{ \exp\left[\frac{1}{T} \left(V_i + \sqrt{k^2 + (m_i^*)^2} - \mu_i \right) + 1 \right] \right\}^{-1}$$

μ_i modified nonrelativistic chemical potential

m_i^* effective mass

V_i, S_i vector and scalar potentials

Idea:

Constrain the vacuum couplings by comparing the description of low-density matter in the virial expansion with the RMF results using an expansion in the powers of fugacities

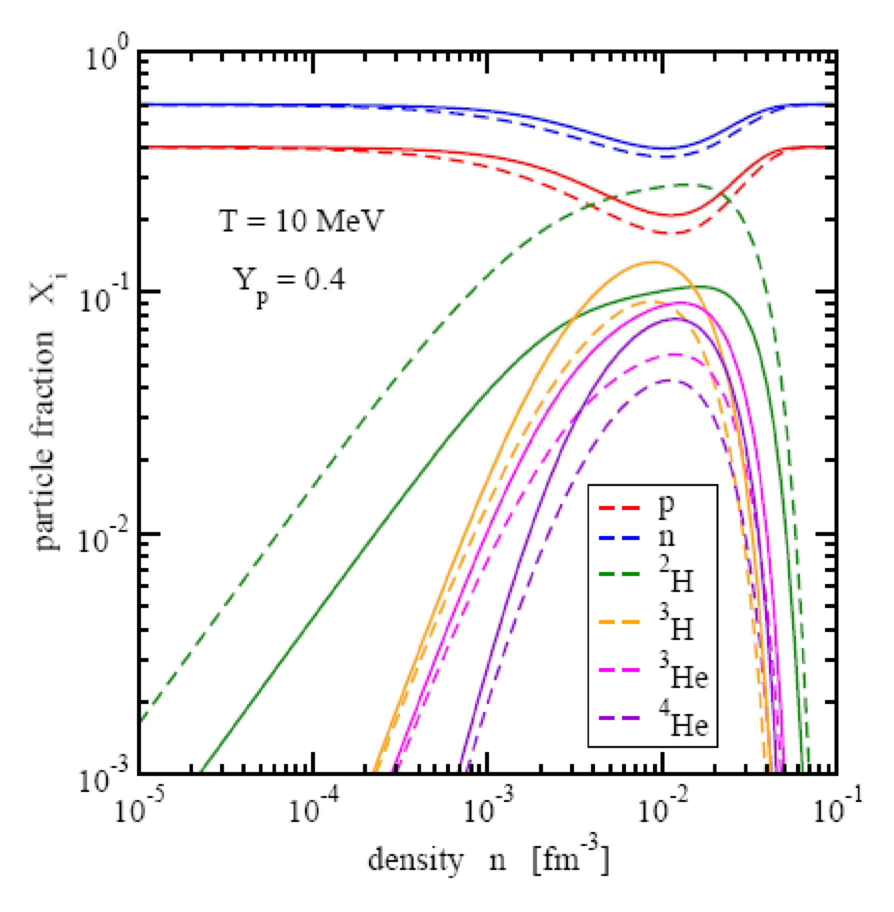
$$z_i = \exp\left(\frac{\mu_i}{T} \right)$$

We perform an expansion in powers of neutron and proton fugacities, leaving only terms linear or quadratic in z_i

Generalized RMF with continuum correlations

- Continuum contributions of nn, pp and np scattering states can be represented effectively by resonances and treated like additional cluster states with $E_{ij}(T)$

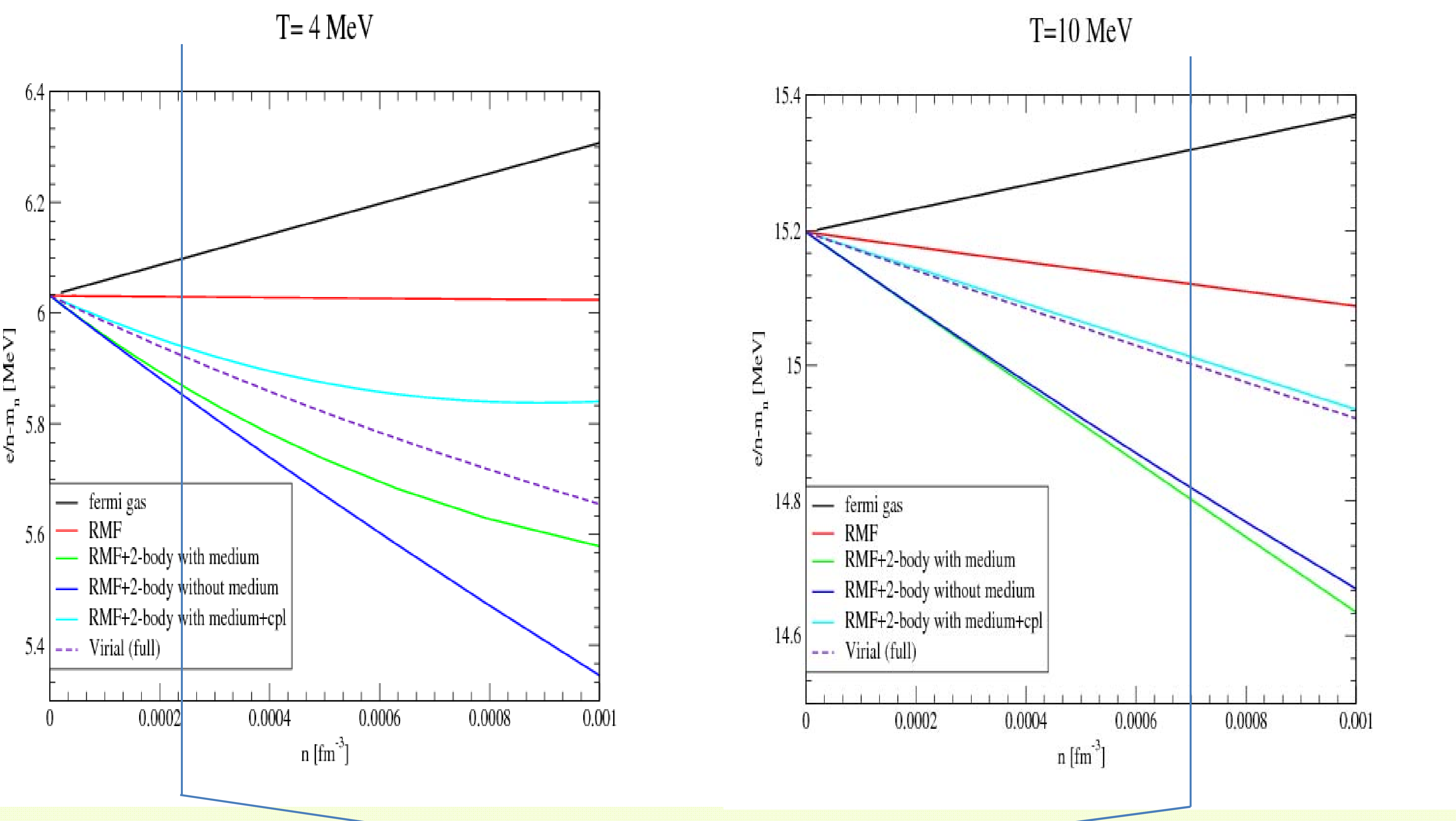
generalized DD-RMF model
(S. Typel et. al, Phys. Rev. C 81 (2010) 065803)
without (dashed) and with (solid) continuum correlations
(without heavy clusters)



- A comparison of the RMF results with the virial expression of the particle densities leads to the definition of $E_{ij}(T)$

- Now we can do the refitting of the parameters to the properties of finite nuclei

Comparison of the Virial EoS with RMF EoS for different temperatures



$$\rho\lambda_i^3 \approx 0.1$$

Cluster formation and dissolution

- Attractive short-range nuclear interaction
- formation of clusters/nuclei
- nucleon-nucleon correlations (treated like additional cluster states)

low-density limit

- finite temperatures: nucleons and two-body correlations dominant
- n+p+α not sufficient
- exact limit: virial EoS
- continuum correlations important
- zero temperature: heavy nuclei
- TF approximation to be corrected or Hartree calculation

high-densities close to saturation

- dissolution of clusters
- generalized DD-RMF model with results from QS/GBU approach

Conclusions and Outlook

Theoretical models of EoS with clusters

- quantum statistical approach (QS)
- generalized relativistic mean-field model (gRMF)
- both thermodynamically consistent
- correct limits at low and high densities

Nuclear matter at low densities

- formation of clusters with medium dependent properties
- inclusion of continuum correlations
- modification of thermodynamical properties/symmetry energies

Future

- improvement of RMF parametrization (low-density limit)
- application to astrophysical models