A Novel Approach to Model Hybrid Stars

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Neutron Stars:

- giant nuclei $\rightarrow$ nuclear constraints
- several times saturation density $\rightarrow$ different particles ($\sim 3.5\rho_0$)
  $\downarrow$
  massive stars!
- fast rotation  
  Negreiros et al. E-print 2011
- high magnetic fields
- ...

- deconfinement
  to quark matter ($\sim 3.7\rho_0$)
SU(3) Non-Linear Sigma Model:


- effective quantum relativistic model → mean field
- describes hadrons interacting via meson exchange (σ, δ, ζ, ω, ρ, φ)
- constructed from symmetry relations → allow it to be chiral invariant → masses from interaction with medium

\[ m_b^* = g_{b\sigma}\sigma + g_{b\delta}\tau_3\delta + g_{b\zeta}\zeta + \delta m_b \]

- chiral symmetry spontaneously broken in nature
- no parity doublets
- pseudo-Goldstone bosons (pseudo-scalar mesons)
Parameter Fitting:

- coupling constants reproduce: standard nuclear constraints, baryon vacuum masses, hyperon potentials, pion and kaon decay constants

- nuclear matter saturation properties
  \( \rho_0 = 0.15 \text{ fm}^{-3}, \ E_{\text{sym}_0} = 29.56 \text{ MeV}, \ L_0 = 88.18 \text{ MeV}, \ p_0 = 4.52 \text{ MeV/fm}^3, \ B = -16.00 \text{ MeV}, \ K_0 = 298 \text{ MeV} \)

- inclusion of the delta meson (scalar-isovector)
  \( E_{\text{sym}_0} = 32.47 \text{ MeV}, \ L_0 = 93.85 \text{ MeV}, \ p_0 = 4.82 \text{ MeV/fm}^3 \)
Inclusion of Gravity (TOV):

- new degrees of freedom make EOS softer (star less massive!)
Extended SU(3) Non-Linear Sigma Model:

- hadronic matter (n, p, Λ, … e, μ)
- quark matter (u, d, s)
- different orders of phase transition
- liquid-gas phase transition
- order parameters σ, Φ
- effective masses
- potential for Φ (deconfinement)

\[ U = (a_0 \, T^4 + a_1 \, μ^4 + a_2 \, T^2 \, μ^2) \phi^2 + a_3 \, T_0^4 \, \ln \left(1 - 6\phi^2 + 8\phi^3 - 3\phi^4\right) \]
Inclusion of Magnetic Field:

- affects density in which phase transition occurs?
- magnetic field in z-direction
  \[ B(\mu_B): \quad B_{\text{surf}} = 69.25 \text{ MeV}^2 = 10^{15} \text{G} \rightarrow B_c \]
- quantization of energy levels (only charged particles)
- anomalous magnetic moment (effect on non-charged particles)

\[ 1 \text{ MeV}^2 = 1.444 \times 10^{13} \text{ G} \]
Phase Transition:
- $\Phi \rightarrow$ deconfinement
- associated with chiral symmetry restoration
- $B$ delays deconfinement
- effect increased by anomalous magnetic moment (dotted line)
- $P_{\text{matter}} \times \epsilon_{\text{matter}}$
- Gibbs construction
- EOS gets stiffer
Population:
- no magnetic field
- \( B = 1 \times 10^6 \text{ MeV}^2 \) 
  \((1.444 \times 10^{19} \text{ G})\)
  with anomalous magnetic moment
- charged particles
- e quark phase
- level thresholds
- enough to change cooling?
Cooling:

- neutrino emission: direct Urca process \((M > 1.2 \, M_\odot)\)
  modified Urca process
  Brehmstrahlung
- photon emission
- same for quarks

"the energy disappears in the nucleus of the supernova as quickly as the money disappeared at that roulette table" George Gamow
Cooling with Magnetic Field:
- 1.1 solar masses
- 1.93 solar masses (maximum mass for $B=0$)
- with anomalous magnetic moment
- cools slower
**TOV:**

- Magnetic field increases star masses (more with anomalous magnetic moment)
- Extra term in pressure?
  \[ T^{\mu\nu} = \frac{1}{4\pi} \begin{pmatrix} \frac{1}{2}B^*^2 & 0 & 0 & 0 \\ 0 & \frac{1}{2}B^*^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}B^*^2 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}B^*^2 \end{pmatrix} \]
- With anomalous magnetic moment
  - \[ \epsilon_{\text{matter}} + B^*^2/8\pi, \ P_{\text{matter}} + B^*^2/24\pi \] (dashed line)
  - \[ \epsilon_{\text{matter}} + B^*^2/8\pi, \ P_{\text{matter}} + B^*^2/8\pi \] (dotted line)
Conclusions:

- chiral SU(3) non linear sigma model consistent with nuclear matter constraints
- extended chiral SU(3) non linear sigma model changes degrees of freedom naturally for high density/temperature
- allows to study high density matter (any temperature)
- reproduces massive stars
- at T=0 high magnetic field delays phase transition
- EOS gets stiffer (different energy distribution)
- even more massive stars
- without cylindrical TOV there are different possibilities
- magnetic field has small effect on cooling (delay)
Our Lagrangian:

\[ L = L_{Kin} + L_{Int} + L_{Sel f} + L_{SB} - U, \]

\[ L_{Int} = -\sum_i \bar{\psi}_i [\gamma_0 (g_i \omega \omega + g_i \phi \phi + g_i \tau_3 \rho) + M_i^*] \psi_i, \]

\[ L_{Sel f} = -\frac{1}{2} (m_\omega^2 \omega^2 + m_\rho^2 \rho^2 + m_{\phi}^2 \phi^2) \]
\[ + g_4 \left( \omega^4 + \frac{\phi^4}{4} + 3\omega^2 \phi^2 + \frac{4\omega^3 \phi}{\sqrt{2}} + \frac{2\omega \phi^3}{\sqrt{2}} \right) \]
\[ + k_0 (\sigma^2 + \zeta^2 + \delta^2) + k_1 (\sigma^2 + \zeta^2 + \delta^2)^2 \]
\[ + k_2 \left( \frac{\sigma^4}{2} + \frac{\delta^4}{2} + 3\sigma^2 \delta^2 + \zeta^4 \right) + k_3 (\sigma^2 - \delta^2) \zeta \]
\[ + k_4 \ln \frac{(\sigma^2 - \delta^2) \zeta}{\sigma_0^2 \zeta_0}, \]

\[ L_{SB} = m_{\pi}^2 f_\pi \sigma + \left( \sqrt{2} m_k^2 f_k - \frac{1}{\sqrt{2}} m_{\pi}^2 f_\pi \right) \zeta, \]
Introduction Magnetic Field:

\[ B^* = B_{surf} + B_c [1 - e^{b_1 \left( \frac{\mu B - 939}{939} \right)^{b_2}}] \]

\[ E^*_{i\nu s} = \sqrt{k_{z_i}^2 + \left( \sqrt{m_i^{*2} + 2\nu |q_i| B^* - s k_i B^*} \right)^2} \]

\[ E^*_{i s} = \sqrt{k_i^2 + \left( m_i^{*2} - s k_i B^* \right)^2} \]

\[ \nu_{max} = \frac{E^*_{i s} + s k_i B^* - m_i^{*}}{2|q_i| B^*} \]

- anomalous magnetic moment has effect on non-charged fermions
  (\( k_i \) determines the coupling strength of baryons with electromagnetic field tensor)
Thermal Balance and Thermal Energy Transport Equation:

\[
\frac{\partial (l e^{2\phi})}{\partial m} = -\frac{1}{\rho \sqrt{1 - 2m/r}} \left( \epsilon_{\nu} e^{2\phi} + c_v \frac{\partial (T e^\phi)}{\partial t} \right)
\]

\[
\frac{\partial (T e^\phi)}{\partial m} = -\frac{(l e^\phi)}{16\pi^2 r^4 \kappa \rho \sqrt{1 - 2m/r}}.
\]

- boundary conditions: luminosity \( l \) vanishes at star's center, surface luminosity determined by mantle temperature and temperature outside the star
- microscopic input: neutrino emissivity \( \epsilon_{\nu}(r,T) \), specific heat \( c_v(r,T) \) and thermal conductivity \( \kappa(r,T) \)
Inclusion of $\Delta$ Resonances:

- Schurhoff et al. APJ 2010
- Oezel Phys. Rev. D 2010

$$r_v = \frac{g_\Delta \omega}{g_N \omega}$$